OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Tasmanian Certificate of Education
External Assessment 2017

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS

(MTM415117)

Pages:	20
Questions:	20
Attachments:	Information Sheet

PART 1

Calculators are NOT allowed to be used

Time: 80 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.
5. The 2017 External Examination Information Sheet for Mathematics Methods can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Understand polynomial, hyperbolic, exponential and logarithmic functions.

Criterion 5 Understand circular functions.

Criterion 6 Use differential calculus in the study of functions.

Criterion 7 Use integral calculus in the study of functions.

Criterion 8 Understand binomial, and normal probability distributions and statistical inference.

BLANK PAGE

Additional Instructions for Candidates

This part of the examination is worth 80 marks in total. Each section is worth 16 marks.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes after examination commencement.

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no workings are required, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks, you are **required to show** relevant working.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

For Marker Use Only

Question 1

The expansion of $(2-3x)^3$ is equal to $ax^3 + 54x^2 + bx + 8$.

Find the values of a and b. (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Question 2

Find the zeros of the function $f(x) = \frac{1}{(x+2)^2} - 3, \ x \neq -2.$ (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section A continues.

For Marker Use Only

Question 3

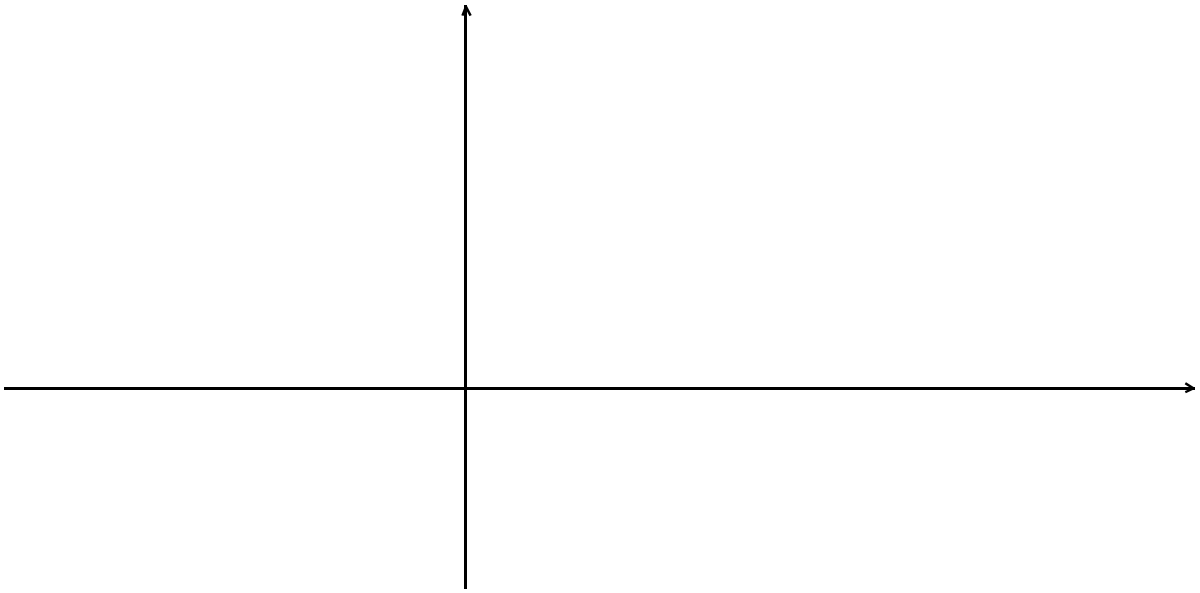
Sketch a graph of the function $y = \log_2(x+4) - 1$, $x \in (-4, \infty)$. Include the zero, y-intercept and asymptote in your sketch. (4 marks)

.....

.....

.....

.....

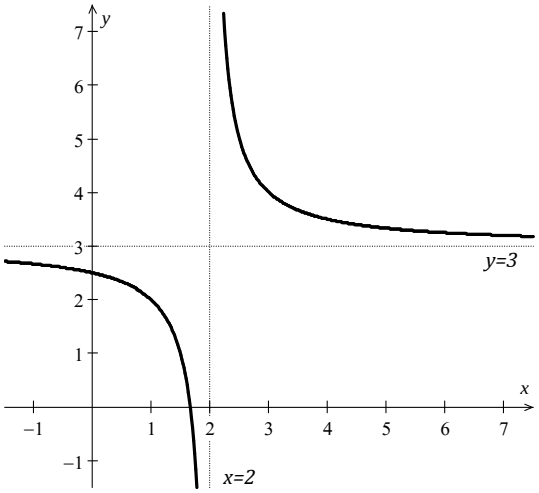


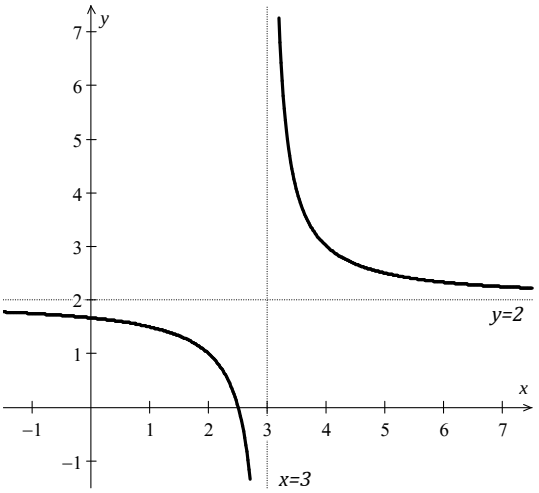
Section A continues.

For Marker Use Only

Question 4

(a) Show that the inverse of the function $f(x) = \dfrac{1}{x-2} + 3$ (pictured on the left) is given by $f^{-1}(x) = \dfrac{1}{x-3} + 2$ (pictured on the right). (2 marks)





.....

.....

.....

.....

.....

.....

(b) A student realises they could generate the function on the right by translating the function $f(x)$ right one unit and down one unit.

They call this new function (also pictured on the right) $g(x)$.

Perform the stated translations to show that $g(x)$ also looks like the function pictured on the right. (2 marks)

.....

.....

.....

.....

Question 4 continues.

Question 4 (continued)

For Marker Use Only

(c) The student notices that with their translations the point $\left(0, \frac{5}{2}\right)$ is transformed to the point $\left(1, \frac{3}{2}\right)$. This confuses the student because they know that $f(0) = \frac{5}{2}$ and $f^{-1}\left(\frac{5}{2}\right) = 0$.

Explain why the translations do not work as the student expected. (2 marks)

.....

.....

.....

.....

Criterion 4 Total

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

For Marker Use Only

Question 5

(a) Simplify $\sin(150^\circ) + \cos\left(120^\circ\right)$. (2 marks)

.....

.....

.....

.....

.....

.....

(b) Simplify $3\sin^2(\theta) + 3\cos^2\left(-\theta\right)$. (3 marks)

.....

.....

.....

.....

.....

.....

Question 6

Given $\tan x = -2$, find $\sin(x)$ given that $\frac{\pi}{2} \le x \le \pi$. (3 marks)

.....

.....

.....

.....

Section B continues.

Section B (continued)

Question 7

For Marker Use Only

(a) Solve $2\sin\left(x-\frac{\pi}{4}\right)=\sqrt{3},\ -\pi\le x\le\pi$. (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

(b) Solve $\sin\left(2\left(x-\frac{\pi}{6}\right)\right)-\sqrt{3}\cos\left(2\left(x-\frac{\pi}{6}\right)\right)=0,\ 0\le x\le 2\pi$. (5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 5 Total

SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

For Marker Use Only

Question 8

Find the derivative of $y = 2e^{3x-1} + x^{e+1}$. (2 marks)

.....

.....

.....

Question 9

Given $y = -\sqrt{x}$, find the value of $\frac{dy}{dx}$ when $x = 4$. (2 marks)

.....

.....

.....

.....

.....

.....

Section C continues.

Section C (continued)

Question 10

For Marker Use Only

A student in class is asked to find the derivative of $f(x) = \ln\left(\frac{x^2+2}{3x-1}\right)$.

(a) Show that by using the **Quotient Rule** the student finds the derivative of $f(x) = \ln\left(\frac{x^2+2}{3x-1}\right)$ to be $f'(x) = \frac{3x^2-2x-6}{(x^2+2)(3x-1)}$. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

(b) The teacher suggests a simpler method using the logarithm law:

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$.

First simplify *f(x)*, then find the derivative $f'(x)$.

Show that the answer is identical to part (a). (4 marks)

.....

.....

.....

.....

.....

.....

.....

Section C continues.

Section C (continued)

Question 11

Find the derivative of $f(x) = 3x^2 - 2$ from first principles. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

For Marker Use Only

Criterion 6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

For Marker Use Only

Question 12

(a) Find $\int \frac{e^{3x+2}}{4} dx$ (2 marks)

.....

.....

.....

(b) Evaluate $\int_3^5 \left(\frac{2}{x} + \frac{3}{x-2} \right) dx$ (2 marks)

.....

.....

.....

.....

Question 13

Differentiate xe^x and use this result to find $\int xe^x dx$. (3 marks)

.....

.....

.....

.....

.....

Section D continues.

Section D (continued)

For Marker Use Only

Question 14

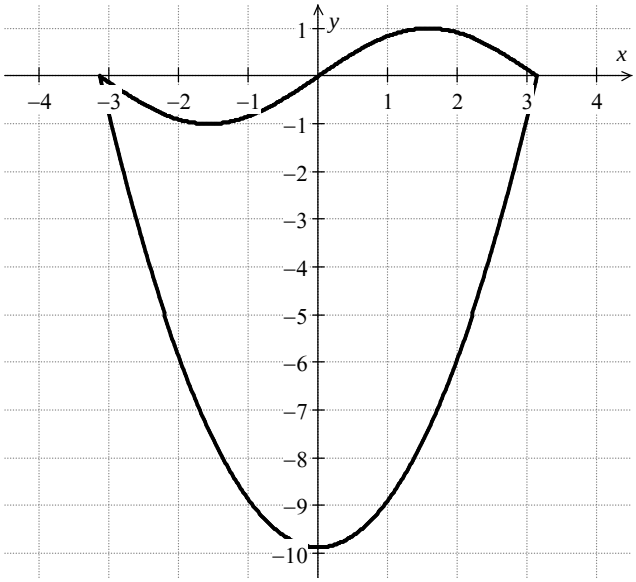
A new company logo is to be made using the two functions

$f(x) = (x - \pi)(x + \pi)$

and

$g(x) = \sin(x),$

where the dimensions are in cm.



Determine the exact value of the area between the two curves. (5 marks)

.....

.....

.....

.....

.....

.....

.....

Question 15

Solve $\int_{-a}^{a} (a^4 - x^4)dx = \frac{800\,000}{5}$, for $a \in \mathbb{R}$. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 7 Total

SECTION E

Answer **ALL** questions in this section.

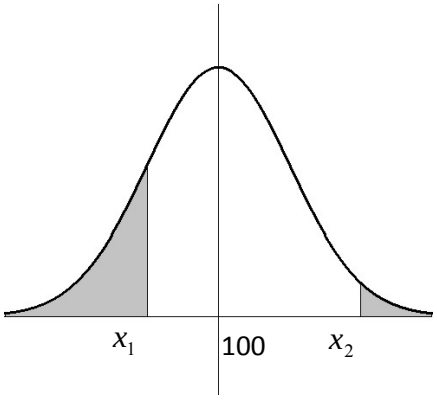
This section assesses **Criterion 8**.

Question 16

For Marker Use Only

A normal distribution has a mean of 100 and a standard deviation of 10.

Find the values x_1 and x_2 given that the proportion of the distribution below x_1 is 16% and the proportion above x_2 is 2.5%. (2 marks)



.....

.....

.....

Section E continues.

For Marker Use Only

Question 17

1000 voters are sampled and 400 prefer Party A in a two-party preferred poll.

This gives a probability estimate of $\hat{p}$ for the proportion of Party A voters in the population.

(a) Write a numerical expression for the mean of $\hat{p}$, $E(\hat{p})$. (1 mark)

.....

.....

(b) Write a numerical expression for the standard deviation of $\hat{p}$, $\sigma_{\hat{p}}$. (1 mark)

.....

.....

Question 18

In a fantasy card game a player has 3 chances to freeze an opponent.

For each chance the probability of succeeding is $\frac{1}{3}$.

Given that each chance is independent of the others, what is the probability of at least one of the chances succeeding? (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section E continues.

For Marker Use Only

Question 19

X is a Binomial Distribution with the number of outcomes, *n*, equal to 9 and the probability, *p*, of success being $\frac{1}{4}$.

Given that $Pr(X = 0) = 0.075$ (to 3 decimal places):

(a) What is $Pr(X > 0)$? (1 mark)

.....

.....

(b) If instead $p = \frac{3}{4}$, what is $Pr(X = 9)$? (2 marks)

.....

.....

.....

.....

Section E continues.

Section E (continued)

Question 20

For Marker Use Only

In a game a four-sided die is rolled twice. The amount that the first roll is greater than the second roll is recorded. The following table is produced:

		Second Roll			
		1	**2**	**3**	**4**
First Roll	**1**	0	-1	-2	-3
	2	1	0	-1	-2
	3	2	1	0	-1
	4	3	2	1	0

The following probability distribution is made based on this data:

x	-3	-2	-1	0	1	2	3
$Pr(X = x)$			$\frac{3}{16}$				

(a) Complete the 2nd row of the table **above**. (1 mark)

(b) What is the value of $E(X)$? (1 mark)

.....

.....

(c) Calculate the standard deviation of the distribution X, $sd(X)$. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 8 Total

BLANK PAGE

OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Tasmanian Certificate of Education
External Assessment 2017

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS

(MTM415117)

Pages:	24
Questions:	14
Attachments:	Information Sheet

PART 2

Calculators are allowed to be used

Time: 100 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 100 minutes in total answering the questions in this booklet.
5. The 2017 External Examination Information Sheet for Mathematics Methods can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Understand polynomial, hyperbolic, exponential and logarithmic functions.

Criterion 5 Understand circular functions.

Criterion 6 Use differential calculus in the study of functions.

Criterion 7 Use integral calculus in the study of functions.

Criterion 8 Understand binomial, and normal probability distributions and statistical inference.

BLANK PAGE

Additional Instructions for Candidates

This part of the examination is worth 100 marks in total. Each section is worth 20 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no workings are required, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks, you are **required to show** relevant working.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

For Marker Use Only

Question 21

Moore's Law said that the number of transistors designers are able to fit in a computer chip doubles every 2 years.

(a) Using the data in the table below find and equation in the form $y = a \times b^x$ that models this situation. Include exact values in your answer. (4 marks)

Years since 1980 (t)	Thousands of transistors (T)
0	50
2	100
6	400

.....

.....

.....

.....

.....

(b) Using your model predict the year when the number of transistors reached 1 million (ie. 1000 thousands). (2 marks)

.....

.....

.....

.....

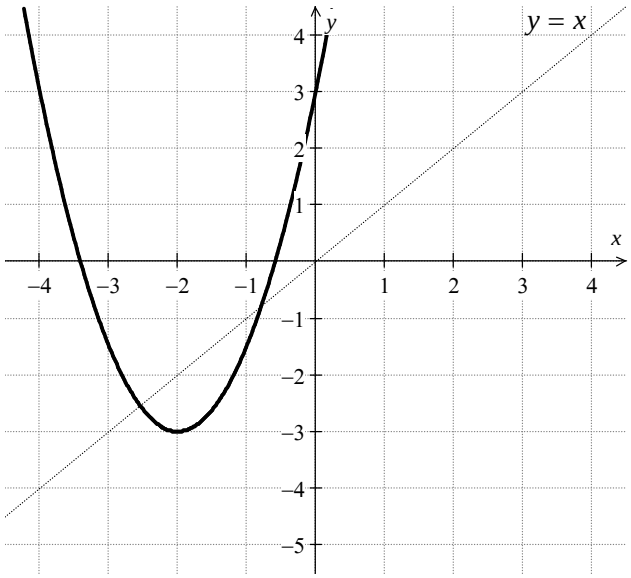
.....

Section A continues.

For Marker Use Only

Question 22

(a) Find an equation to represent the parabola $p(x)$ pictured below. (3 marks)



.....

.....

.....

.....

.....

.....

(b) The domain of the function $p(x)$ is restricted to $(-\infty, -2]$ so that an inverse function $p^{-1}(x)$ will exist.

Sketch the graph of $p^{-1}(x)$ on the same set of axes. (3 marks)

Section A continues.

Section A (continued)

Question 23

For Marker Use Only

Two functions are defined by $f(x) = \ln(x-5)+2$ and $g(x) = e^{2x-1}+4$.

(a) $(f \circ g)(x) = f(g(x))$ will only be defined when the range of $g(x)$ is a subset of the domain of $f(x)$.

Hence calculate the domain of $f(g(x))$. (4 marks)

.....

.....

.....

.....

.....

.....

.....

(b) Now consider $g(f(x))$ using $f(x)$ and $g(x)$ as defined above.

Calculate the domain of $g(f(x))$ and find an expression for $g(f(x))$. (4 marks)

.....

.....

.....

.....

.....

.....

.....

Criterion 4 Total

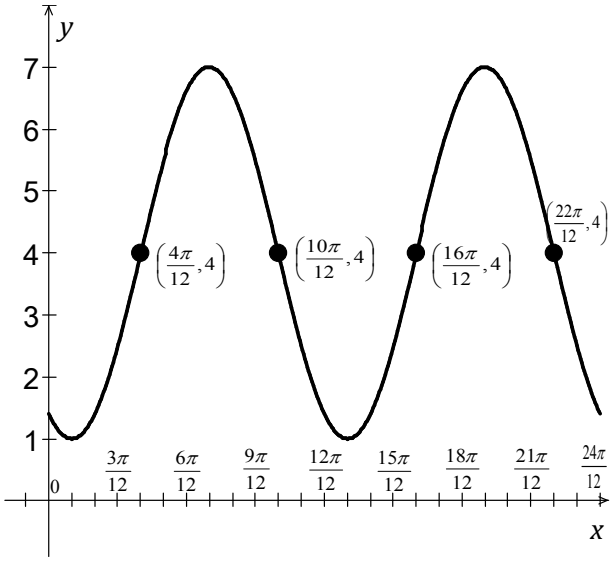
SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Question 24

For Marker Use Only



Find an equation to represent the picture above for $0 \leq x \leq 2\pi$. (5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Section B continues.

For Marker Use Only

Question 25

A miniature windmill with a radius of 1m is revolving anti-clockwise at the speed of 360º every 30 seconds.

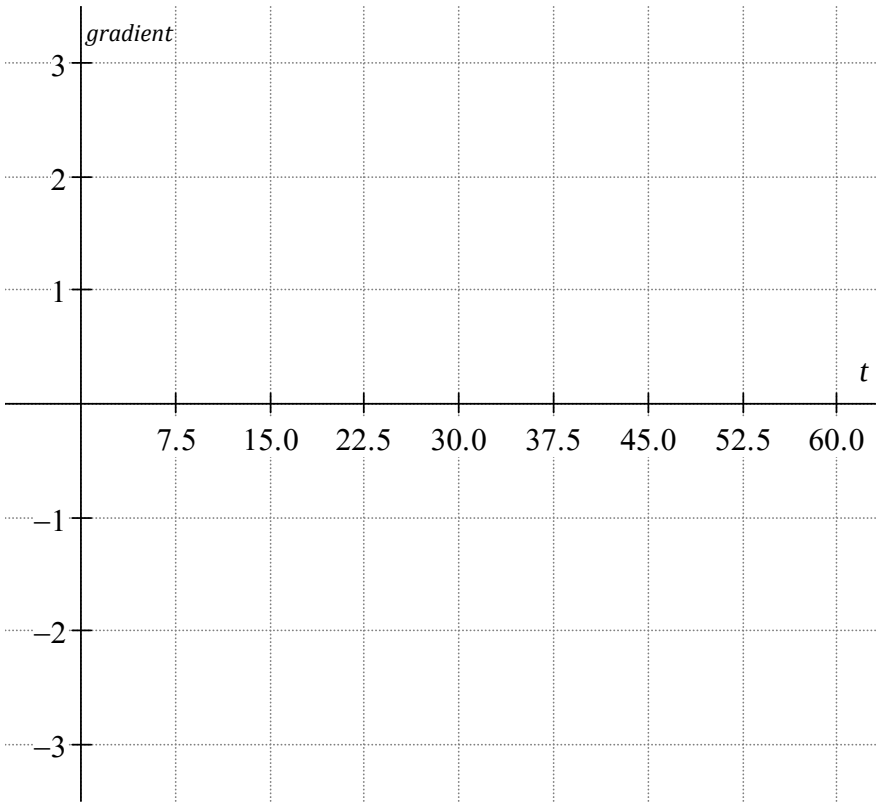
One arm of the windmill has a gradient of 0 at t = 0.

The gradient of the arm over time can be modelled by a tangent function of the form:

$$y = a\tan(nt)$$

(a) Sketch the graph of the tangent function for two complete revolutions (1 minute).

(4 marks)



Section B continues.

Section B (continued)

For Marker Use Only

(b) Show that the equation to represent the gradient of the arm of the windmill, for $0 \le t \le 60$, can be given by $gradient = \tan\left(\frac{\pi}{15}t\right)$. (3 marks)

.....

.....

.....

.....

.....

(c) A fly sits on the arm of the windmill at t = 0. It wants to ride on the arm but can only stay on the arm while the gradient of the arm is between -3 and 3. It also does not want to travel upside down.

For what intervals during the one minute can the fly stay on the arm? (3 marks)

.....

.....

.....

.....

.....

.....

(d) After the first minute the windmill begins to travel twice as fast.

(i) Describe the equation of the gradient of the arm for the second minute (ie. $60 \le t \le 120$ seconds). (2 marks)

.....

.....

(ii) How much time will the fly spend on the arm in the second minute? (3 marks)

.....

.....

.....

.....

.....

Criterion 5 Total

SECTION C

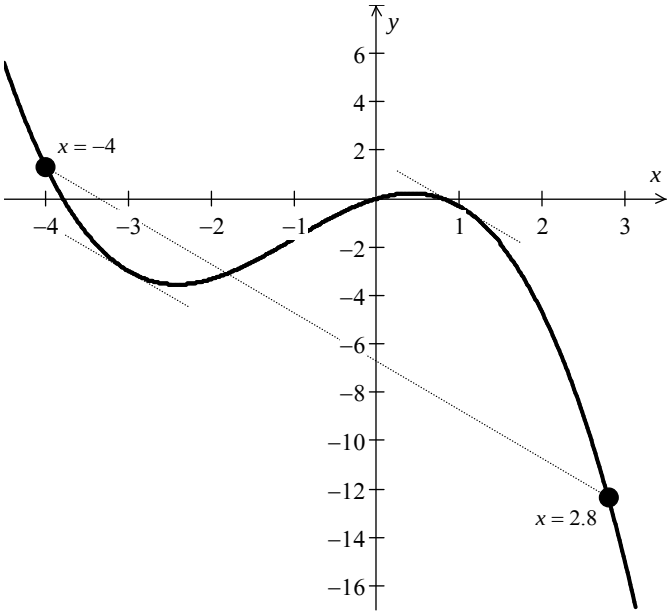
Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

For Marker Use Only

Question 26

For the function $f(x) = -\frac{1}{3}x^3 - x^2 + x$ over the interval $[-4, 2.8]$ (pictured below) there are two places in the interval where the slope of the tangent to the curve is the same as the average rate of change.



(a) Show that the average rate of change for $f(x)$ over the interval [-4,2.8] is equal to -2.0 (to one decimal place). (3 marks)

.....

.....

.....

.....

.....

Section C continues.

For Marker Use Only

(b) Assuming the average rate of change is exactly -2, find the two tangents to the curve $f(x)$ where the slope of the tangent is also -2. Give exact values in your answer.

(4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Section C continues.

For Marker Use Only

Question 27

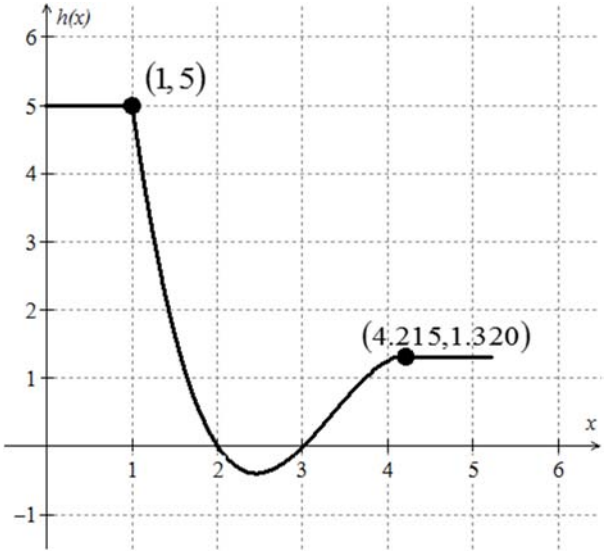
A new super-slide is to be constructed for children with side-view given by:

$$h(x)=\begin{cases} 3, & 0\le x\le 1 \\ -\frac{5}{8}(x-2)(x-3)(x-5), & 1\le x\le 4.215 \\ 1.320 & 4.215\le x\le 5.215 \end{cases}$$

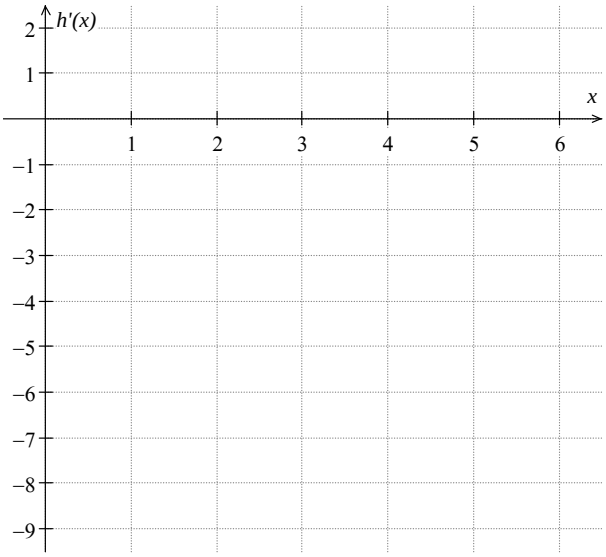
The slide is designed so that at $x=4.215$ the slide makes a smooth horizontal connection with the platform.

(a) Given the sketch of $h(x)$ below, sketch the derivative of the function $h(x)$ in the axes provided, labelling all stationary points and marking clearly anywhere the derivative is not defined. (4 marks)

Sketch of $h(x)$:



Sketch of $h'(x)$:



Question 27 continues.

(b) Parents are concerned about the steepness of the slide, both downwards and upwards. Find both the largest positive and negative gradients of the slide to 3 decimal places and justify your answers. (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

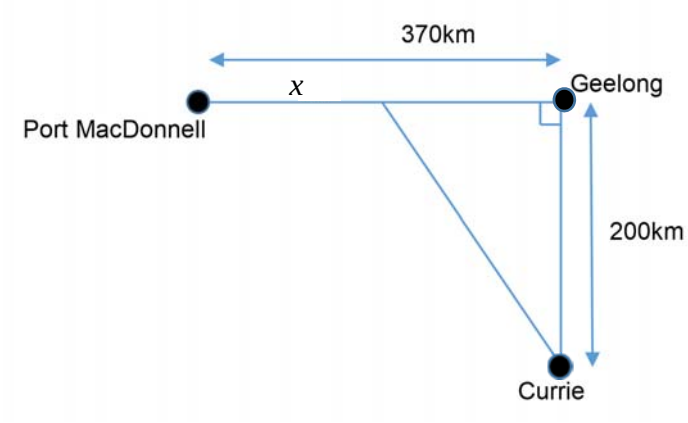
.....

Section C continues.

Section C (continued)

Question 28

For Marker Use Only



Fish are brought to Currie at King Island and need to be processed as quickly as possible.

The processing plant is at Port MacDonnell in Victoria, 370 km west of Geelong.

A port is to be built x km from Port MacDonnell with the fish transported by cargo ship at 50km/h, then loaded on to trucks at the new port and taken at 80km/h to Port MacDonnell.

(a) Using $time = \frac{distance}{speed}$ show that the total time is given by:

$$t = \frac{x}{80} + \frac{\sqrt{200^2 + (370 - x)^2}}{50}$$

(2 marks)

.....

.....

.....

.....

(b) Find the distance x from Port MacDonnell for the best place for the new port to be built so that the fish can arrive as soon as possible. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

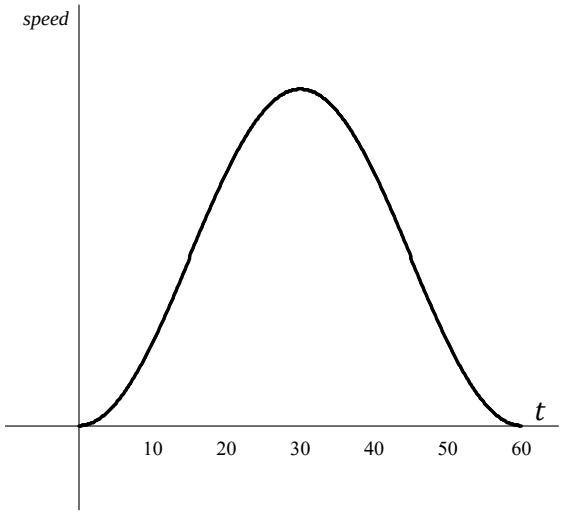
For Marker Use Only

Question 29

The speed of a car in metres per second is recorded as it travels without stopping from one set of traffic lights to the next.

$$speed = a\left(-\cos\left(\frac{2\pi}{60}t\right)+1\right) \text{ ms}^{-1}$$

for $t \in [0, 60]$ in seconds and $a \in \mathbb{R}$.



(a) Show that the distance travelled by the car is given by $60a$ metres. (3 marks)

.....

.....

.....

.....

(b) Given that the distance between the two traffic lights is 400m find the value of a. (2 marks)

.....

.....

.....

.....

Section D continues.

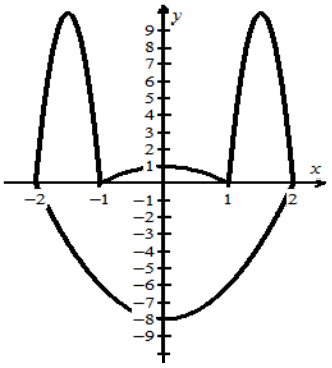
Section D (continued)

For Marker Use Only

Question 30

The outline of a rabbit can be formed by the following four parabolas:

$f_1(x) = 2(x-2)(x+2)$, for $-2 \le x \le 2$,
$f_2(x) = -40(x+1)(x+2)$, for $-2 \le x \le -1$,
$f_3(x) = -(x-1)(x+1)$, for $-1 \le x \le 1$,
and $f_4(x) = -40(x-1)(x-2)$, for $1 \le x \le 2$.



p

Find the exact area of the rabbit by finding the area enclosed by the four parabolas.

(5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Section D continues.

Section D (continued)

For Marker Use Only

Question 31

The function has derivative given by $f'(x) = 6x^2 + ax + b$, for $a, b \in \mathbb{R}$.

The function has tangent equations

$y = 39x - 42$ at $x = 2$, and
$y = -3x + 6$ at $x = -1$.

By finding the values of *a* and *b*, determine the function $f(x)$. (5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Section D continues.

Section D (continued)

Question 32

The graph of a function has a gradient given by $\frac{dy}{dx} = (3x+1)(x-1)$ and has a minimum at a point with $y = 2$.

Find the equation of the function. (5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

For Marker Use Only

Criterion 7 Total

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

For Marker Use Only

Question 33

The length of bananas grown on a plantation is normally distributed with a mean of 20 cm and a standard deviation of 5 cm.

When picked, bananas with a length of less than 12 cm are deemed too short to sell and are discarded.

What percentage of bananas is discarded? (3 marks)

.....

.....

.....

.....

.....

Question 34

The length of nails produced by a machine are normally distributed with a mean of 80 mm and a standard deviation of σ mm.

Given that the longest 15% of nails have a length of 83 mm or greater, find the value of σ to two decimal places. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section E continues.

For Marker Use Only

Question 35

(a) In a board game 4 six-sided dice are rolled and a 'hit' is scored if at least one of the dice rolls a 1.

What is the probability of a hit being scored? (3 marks)

.....

.....

.....

.....

.....

.....

(b) What is the probability of at least three hits occurring from five sets of rolls? (3 marks)

.....

.....

.....

.....

.....

.....

(c) Will the probability of rolling at least six hits from ten sets of rolls be different from the previous answer? Explain why. (3 marks)

.....

.....

.....

.....

.....

.....

Section E continues.

Section E (continued)

Question 36

For Marker Use Only

(a) A company is conducting election polling and surveys 1000 people over the phone.

They find that 45% of people support Party A and the remainder support Party B.

If they wish to have a 95% level of confidence in their result, what margin of error can they quote in their poll results? (2 marks)

.....

.....

.....

.....

.....

.....

.....

.....

(b) The company would actually like a margin of error of 1% instead (still with a 95% level of confidence in their result).

How many people should they survey instead to obtain this margin of error? (2 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 8 Total

BLANK PAGE

BLANK PAGE