OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Tasmanian Certificate of Education
External Assessment 2018

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS

(MTM415117)

Pages:	20
Questions:	18
Attachments:	Information Sheet

PART 1

Calculators are NOT allowed to be used

Time: 80 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.
5. The 2018 External Examination Information Sheet for Mathematics Methods can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Understand polynomial, hyperbolic, exponential and logarithmic functions.

Criterion 5 Understand circular functions.

Criterion 6 Use differential calculus in the study of functions.

Criterion 7 Use integral calculus in the study of functions.

Criterion 8 Understand binomial, and normal probability distributions and statistical inference.

Additional Instructions for Candidates

This part of the examination is worth 80 marks in total. Each section is worth 16 marks.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected 80 minutes after examination commencement.

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no workings are required, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks, you are **required to show** relevant working.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Section A Marks = 16

Question 1

For Marker Use Only

$(ax + b)^3$ is equal to $8x^3 - 12x^2 + 6x - 1$. Find the values of a and b. (2 marks)

.....

.....

.....

.....

.....

.....

Question 2

A polynomial is given by $P(x) = ax^4 - 2x^3 + bx - 3$, where $a, b \in \mathbb{R}$.
When $P(x)$ is divided by $(x + 1)$ the remainder is 5.
When $P(x)$ is divided by $(x - 1)$ the remainder is -3.

Find a and b. (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section A continues.

For Marker Use Only

Question 3

Find the zeroes of the truncus function $f(x) = \frac{1}{(x-6)^2} - 4,\ x \neq 6.$ (3 marks)

.....

.....

.....

.....

.....

.....

Question 4

Solve $\mathbf{2 \times 16^x + 3 \times 4^x = 2}$ using the substitution $\boldsymbol{A = 4^x}$. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

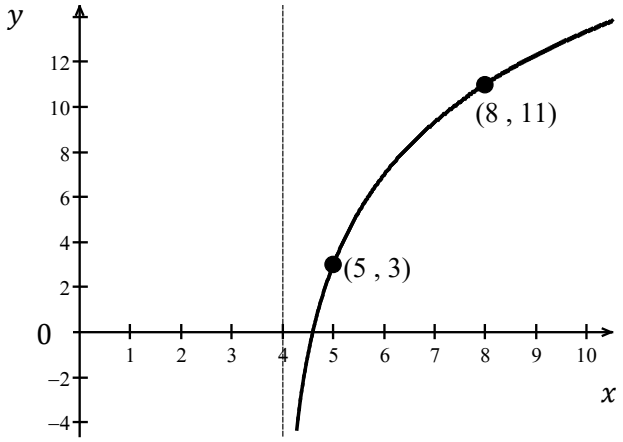
Section A continues.

Section A (continued)

For Marker Use Only

Question 5

A logarithmic equation has the form $y = a\log_2(x - h) + k$ and is pictured below.



(a) What is the domain and range of the logarithmic equation? (1 mark)

Domain: .....

Range: .....

(b) Find the values of a, h and k. (3 marks)

.....

.....

.....

.....

.....

.....

.....

Criterion 4 Total

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B Marks = 16

Question 6

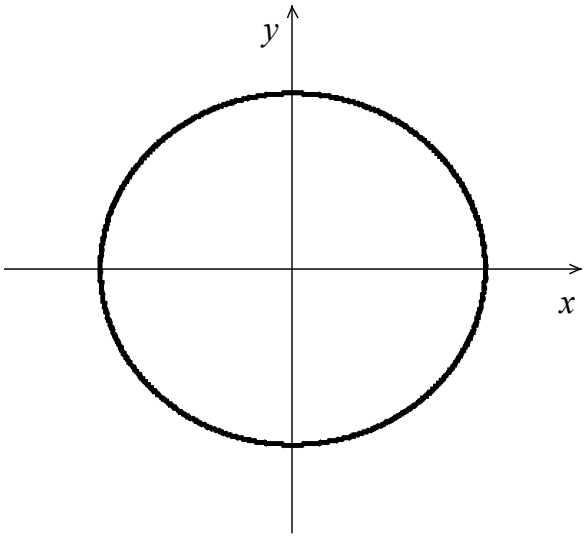
For Marker Use Only

(a) Convert $\frac{5\pi}{12}$ radians into degrees. (1 mark)

.....

.....

(b) Identify on the diagram where an angle of $\frac{5\pi}{12}$ will be. (1 mark)



Question 7

Given that $\cos\theta = -\frac{1}{4}$ and $\frac{\pi}{2} < \theta < \pi$, evaluate $\sin\theta$ and $\tan\theta$. (3 marks)

.....

.....

.....

.....

.....

.....

Section B continues.

Section B (continued)

For Marker Use Only

Question 8

Solve $\tan\left(x - \frac{\pi}{4}\right) = 1$ for $x \in [\pi, 3\pi]$ (3 marks)

.....

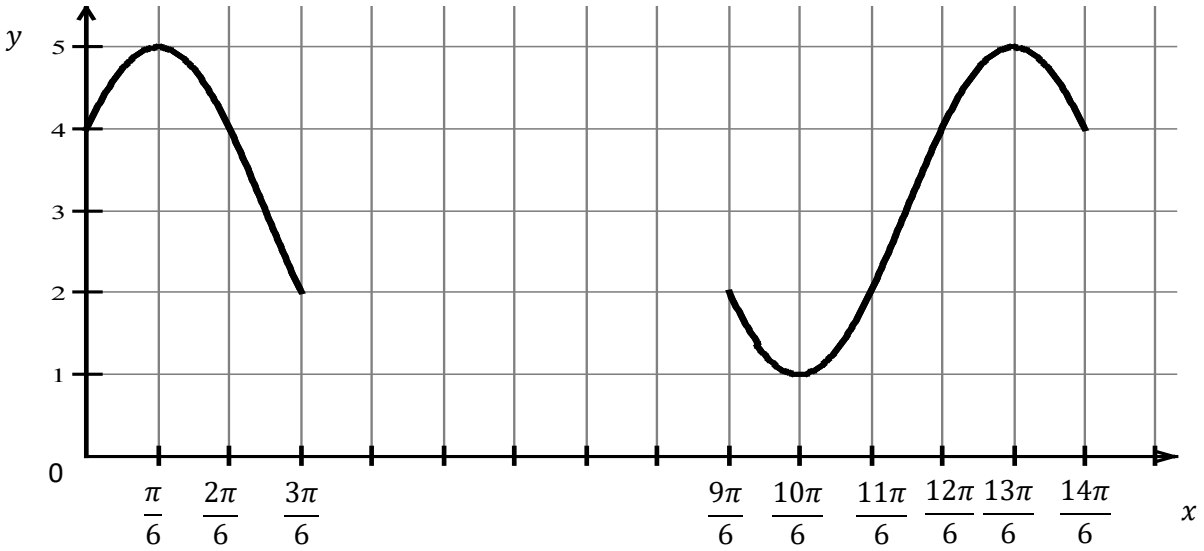
.....

.....

.....

Question 9

Below is a partly drawn graph from an equation of the form $y = a\cos n(x + b) + c$



(a) Complete the sketch of the function, labelling the missing minimum and maximum points. (3 marks)

(b) Find the values of:

(i) a ..... (1 mark)

(ii) c ..... (1 mark)

(iii) n ..... (1 mark)

(iv) b ..... (1 mark)

(c) Check your answers to (b) by calculating y when $x = \frac{\pi}{6}$. (1 mark)

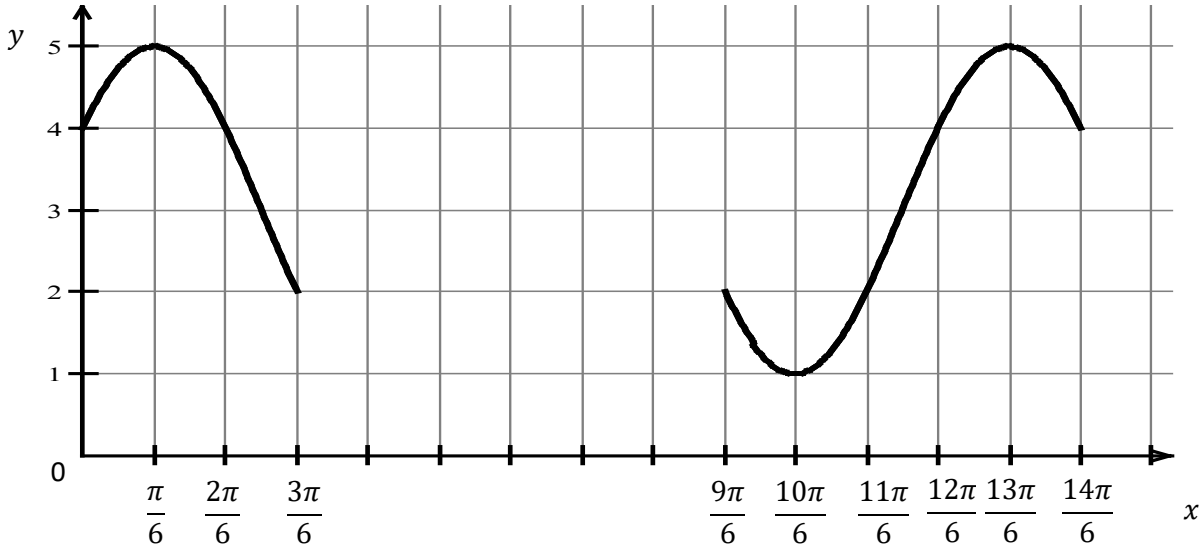
.....

.....

Criterion 5 Total

SPARE DIAGRAM

Question 9



BLANK PAGE

SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C Marks = 16

Question 10

For Marker Use Only

Evaluate the following limits:

(a) $\lim_{x \to -2} \frac{3x+1}{2x-5}$ (1 mark)

.....

.....

(b) $\lim_{x \to 7} \frac{(x-7)^2}{(3x-7)(x-7)}$ (2 marks)

.....

.....

.....

Section C continues.

For Marker Use Only

Question 11

Determine the derivatives of (no simplification required):

(a) $f(x) = 3x^2\sin(x)$ (2 marks)

.....

.....

.....

.....

(b) $y = e^{4\ln(2x)}$ (2 marks)

.....

.....

.....

.....

(c) $g(x) = \frac{(4-x)}{(2x+1)^2}$ (2 marks)

.....

.....

.....

.....

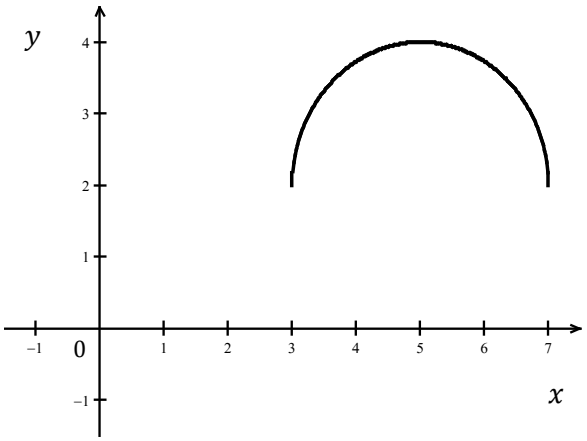
Section C continues.

Section C (continued)

Question 12

The curve shown on the graph can be represented by the function:

$$y = \sqrt{4 - (x - 5)^2} + 2.$$



(a) Find the gradient of the tangent to the curve at $x = 6$. (3 marks)

.....

.....

.....

.....

.....

.....

(b) Hence show that the equation of the normal to the curve at $x = 6$ is given by:

$y = \sqrt{3}x - 5\sqrt{3} + 2$ (3 marks)

.....

.....

.....

.....

.....

.....

(c) Show that the normal passes through the point $(\mathbf{5}, \mathbf{2})$. (1 marks)

.....

.....

For Marker Use Only

Criterion 6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D Marks = 16

Question 13

For Marker Use Only

Evaluate the following integrals:

(a) (2 marks)

$$\int \frac{cos(3x+1)}{2} dx$$

.....

.....

.....

(b) (3 marks)

$$\int_1^3 \left(3x^2 + 7x + \frac{3}{x}\right) dx$$

.....

.....

.....

.....

.....

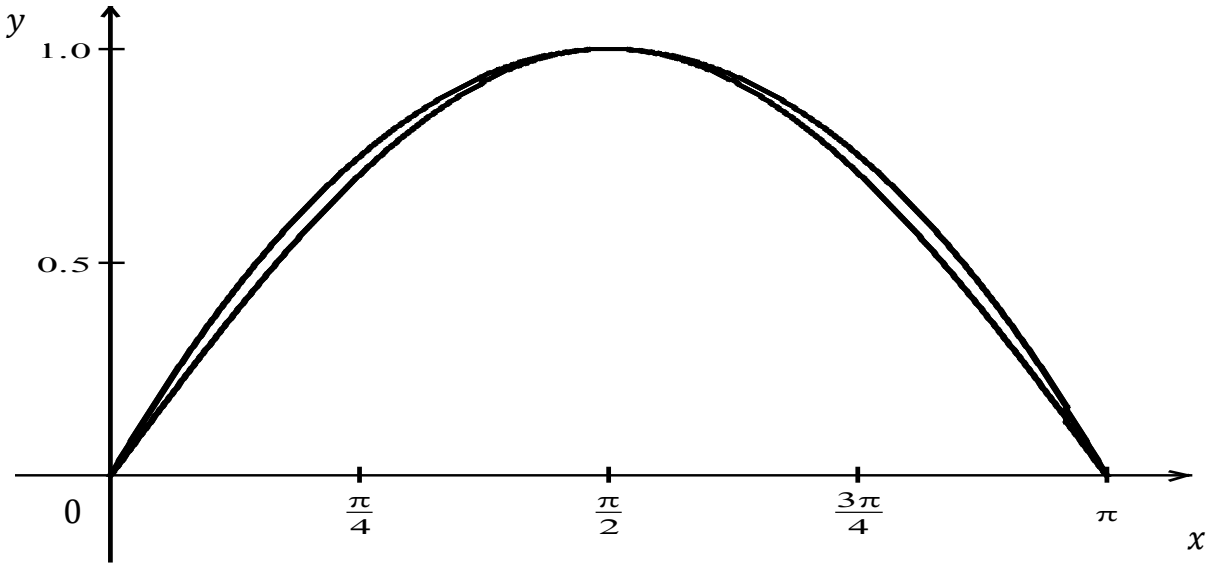
.....

Section D continues.

For Marker Use Only

Question 14

The graph of $f(x) = \sin(x)$ can be modelled very closely by the parabola $g(x) = \frac{4}{\pi^2}x(\pi - x)$ over the interval $[0\ , \pi]$ as shown on the graph below.



(a) Show that $g\left(\frac{\pi}{4}\right) = \frac{3}{4}$. (1 mark)

.....

.....

(b) Since $\frac{3}{4} > \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ then $g(x)$ is the uppermost function.
Find the exact area between the two functions over the interval $[0\ , \pi]$. (4 marks)

.....

.....

.....

.....

.....

.....

Section D continues.

Section D (continued)

For Marker Use Only

Question 15

(a) Show that the derivative of $\frac{xlnx-x}{ln2}$ is equal to $\frac{lnx}{ln2}$. (2 marks)

.....

.....

.....

.....

.....

.....

(b) Using your result in part (a) show that (4 marks)

$$\int_2^4 \left(\frac{lnx}{ln2} - 3\right) dx = -\frac{2}{ln2}$$

.....

.....

.....

.....

.....

.....

.....

Criterion 7 Total

SECTION E

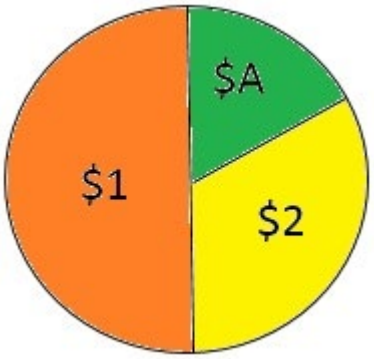
Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E Marks = 16

Question 16

In a casino game, X is the winnings paid out when the wheel pictured opposite is spun.



The probabilities of each payout are shown below:

x	1	2	A
$P\ (X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	k

(a) Find the value of k. (1 mark)

.....

.....

(b) Given that the expected payout, $E(X)$, is to be \$2, find the value of A. (2 marks)

.....

.....

.....

.....

(c) Find the variance of the payout, $\mathrm{var}(X)$. (3 marks)

.....

.....

.....

.....

Section E, Question 16 continues.

Section E, Question 16 (continued)

For Marker Use Only

For Marker Use Only

(d) The casino is told that the expected return of the game from 100 plays will be $E(100X - 300)$.
Find the amount of the expected return to the casino. (2 marks)

.....

.....

.....

.....

Question 17

In a phone poll of 1000 people it is found that 400 answer "yes" to a survey question.

(a) What is the proportion in the sample, $\hat{p}$, who would answer "yes" to the survey question? (1 mark)

.....

.....

(b) Write an expression for the 95% confidence interval for the proportion of people who answered "yes". (2 marks)

.....

.....

(c) By what factor will the margin of error for the 95% confidence interval change if 2000 people were sampled instead, and 800 answered "yes". (2 marks)

.....

.....

.....

.....

Section E continues.

Section E (continued)

Question 18

For Marker Use Only

X is a normal distribution with mean 100 and standard deviation 10.

For the shaded area above the following probability statement can be made:

$$P(100 \leq X \leq 105) = 0.19$$

Write probability statements for this distribution that represent:

(a) 0.31 (1 mark)

.....

.....

(b) 0.69 (1 mark)

.....

.....

(c) 0.38 (1 mark)

.....

.....

Criterion 8 Total

BLANK PAGE

OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Tasmanian Certificate of Education
External Assessment 2018

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS

(MTM415117)

Pages:	24
Questions:	19
Attachments:	Information Sheet

PART 2

Calculators are allowed to be used

Time: 100 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 100 minutes in total answering the questions in this booklet.
5. The 2018 External Examination Information Sheet for Mathematics Methods can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Understand polynomial, hyperbolic, exponential and logarithmic functions.

Criterion 5 Understand circular functions.

Criterion 6 Use differential calculus in the study of functions.

Criterion 7 Use integral calculus in the study of functions.

Criterion 8 Understand binomial, and normal probability distributions and statistical inference.

Additional Instructions for Candidates

This part of the examination is worth 100 marks in total. Each section is worth 20 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected 80 minutes after examination commencement (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no workings are required, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks, you are **required to show** relevant working.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Section A Marks = 20

Question 19

For Marker Use Only

A science student is studying the relationship between the pressure and volume of a gas. The following is a sample of the student's results:

Pressure P (mm Hg)	Volume V (mL)
944	24.9
890	26.4

(a) The data can be modelled in the form $P = \frac{k}{V}$ where k is a constant.
Using this data, find the average value of k. (2 marks)

.....

.....

.....

.....

(b) Predict the volume when the pressure of the gas is 500 mm Hg. (2 marks)

.....

.....

.....

.....

Section A continues.

For Marker Use Only

Question 20

(a) The function $y = e^x$ is translated right by 3 units and up by 1 unit to give the function $f(x)$. Write down the equation of $f(x)$. (1 mark)

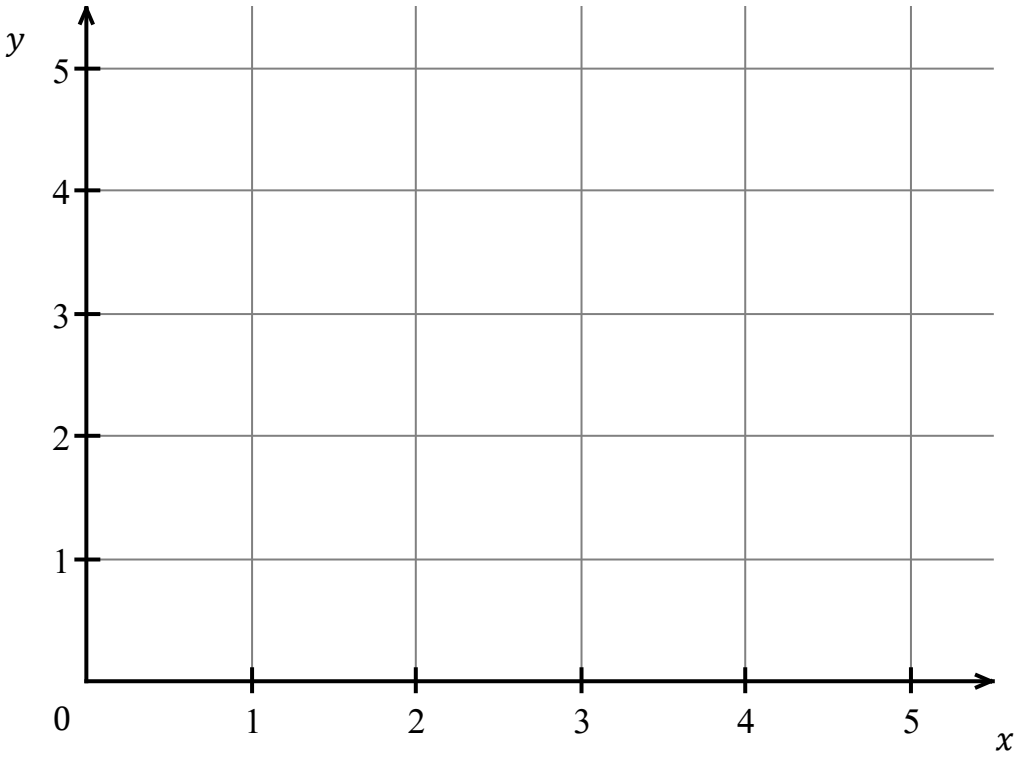
.....

.....

(b) Sketch the graph of $f(x)$ on the axes below.

Include the asymptote and the y-intercept.

Label the point where $x = 3$. (3 marks)



(c) Sketch the inverse on the same set of axes above.
Algebraically find the inverse function $f^{-1}(x)$. (3 marks)

.....

.....

.....

.....

.....

.....

Section A (Question 20) continues.

For Marker Use Only

(d) State the transformations required to transform the equation

$y = \ln(x)$ into $f^{-1}(x)$. (1 mark)

.....

.....

(e) An airplane wing is to be made from the area between these two curves.
State the restricted domain of $f(x)$ that forms the wing.
Give your answer to 2 decimal places. (1 mark)

.....

.....

.....

.....

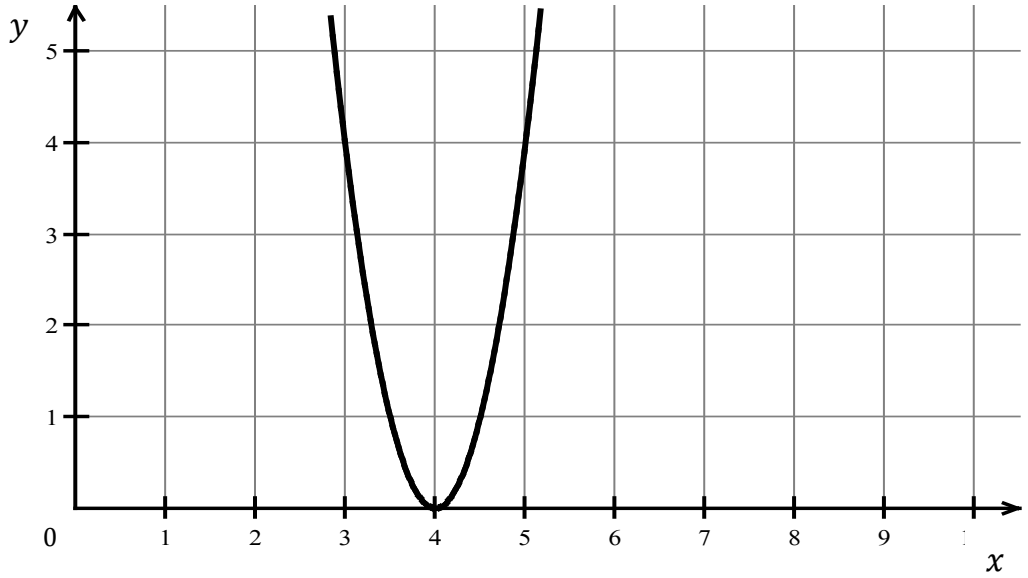
Section A continues

Section A (continued)

Question 21

For Marker Use Only

The sketch of $f(x) = (2x - 8)^2$ is given below.



(a) On the same axes, sketch the graph after a dilation of factor $\frac{1}{2}$ in the x-direction has been applied. (2 marks)

(b) The formula sheet says that a dilation in the direction of the x-axis by a factor of $\frac{1}{2}$ is $f(x) \to f(2x)$.

This dilation can also be determined using the function $g(x) = 2x$ and finding $f(g(x))$. Find $f(g(x))$ and express it in the form $y = a(x - h)^2$. (3 marks)

.....

.....

.....

.....

.....

(c) Using your answer to (b) show that the point on the original graph $(5,\ 4)$ correctly maps to the dilated point $\left(\frac{5}{2},\ 4\right)$. (2 marks)

.....

.....

.....

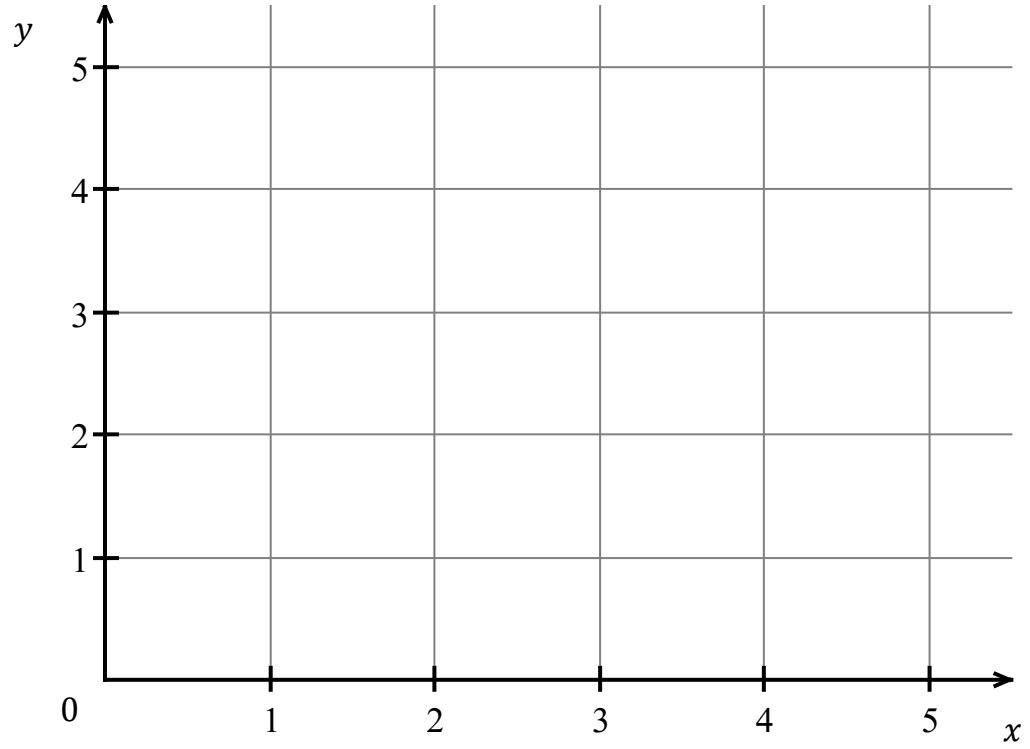
.....

Criterion 4 Total

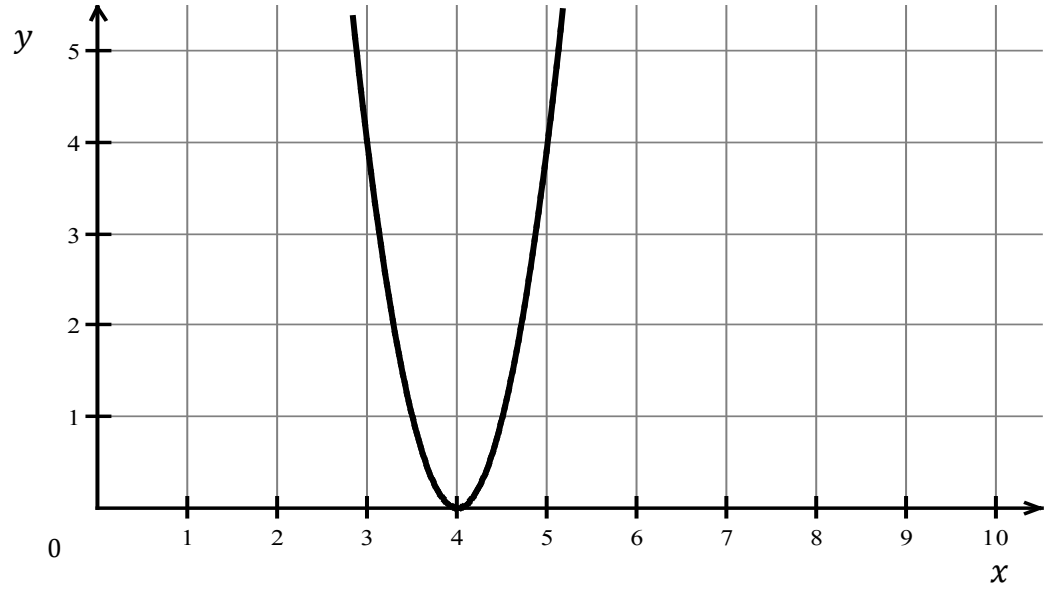
SPARE DIAGRAMS

Section A

Question 20 (b)



Question 21 (a)



SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B Marks = 20.

Question 22

For Marker Use Only

Solve $2\cos\left(2\left(x-\frac{\pi}{6}\right)\right)=\sqrt{2}$, for $0\leq x\leq 2\pi$.

Show all algebraic working in your answer. (6 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

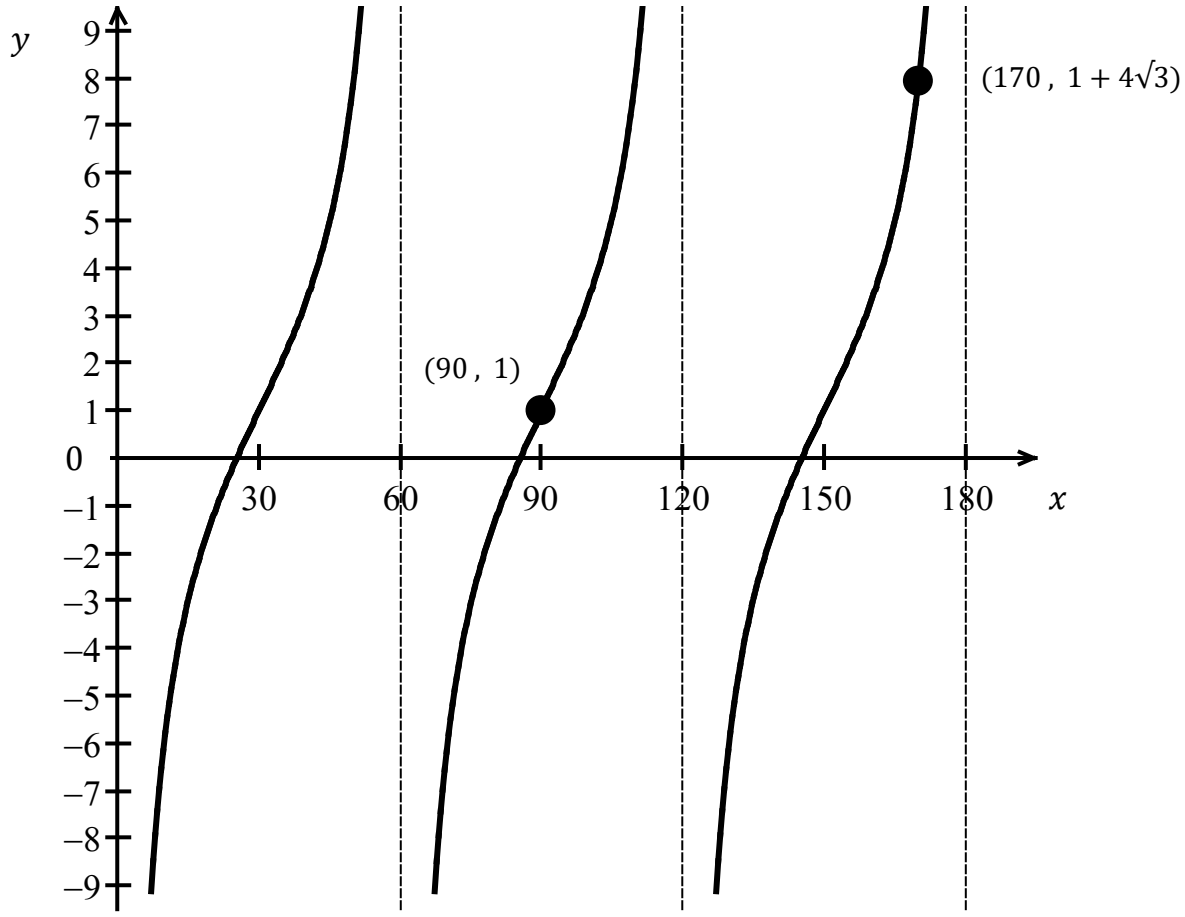
Section B continues.

Section B (continued)

Question 23

The function below can be modelled by an equation of the form

$y = a\ tan\big(n(x + b)\big) + c$, where x is measured in degrees.



Find the values of a, n, b and c.

(a) c ..... (1 mark)

(b) n ..... (1 mark)

(c) b ..... (1 mark)

(d) a .....

.....

.....

.....

..... (2 marks)

Section B continues.

For Marker Use Only

Section B (continued)

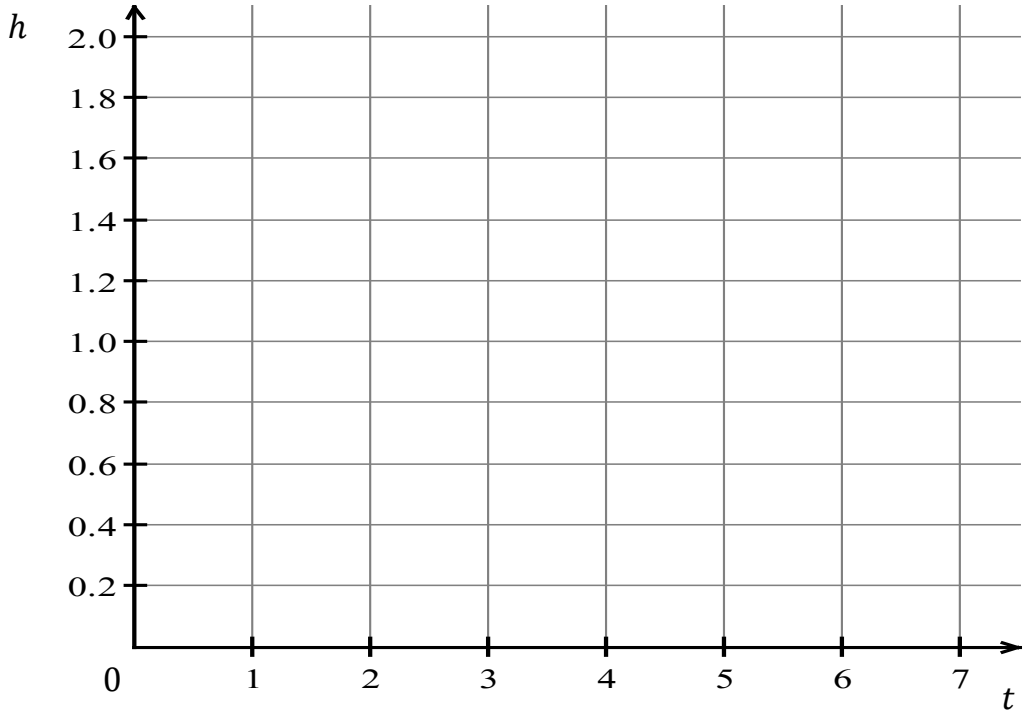
For Marker Use Only

Question 24

The height of the middle of a skipping rope can be modelled by the equation

$h = sin(\frac{\pi}{2}t) + 1$, for $0 \leq t \leq 7$ where *t* is in seconds and h is in metres.

(a) Sketch the graph of h for $0 \leq t \leq 7$. (4 marks)



.....

.....

(b) A camera is set up to film the middle of the rope when it is at the bottom of its rotation. If the length of the filming is 1 second, find the value k for which $\sin(\frac{\pi}{2}t)+1 \leq k$ during the filming. (2 marks)

.....

.....

.....

(c) The start time is changed so that the height of the middle of the skipping rope is below k between 4 and 5 seconds.

Find a value of c so that $\sin\left(\frac{\pi}{2}t + c\right) + 1 \leq k$ for $4 \leq t \leq 5$. (3 marks)

.....

.....

.....

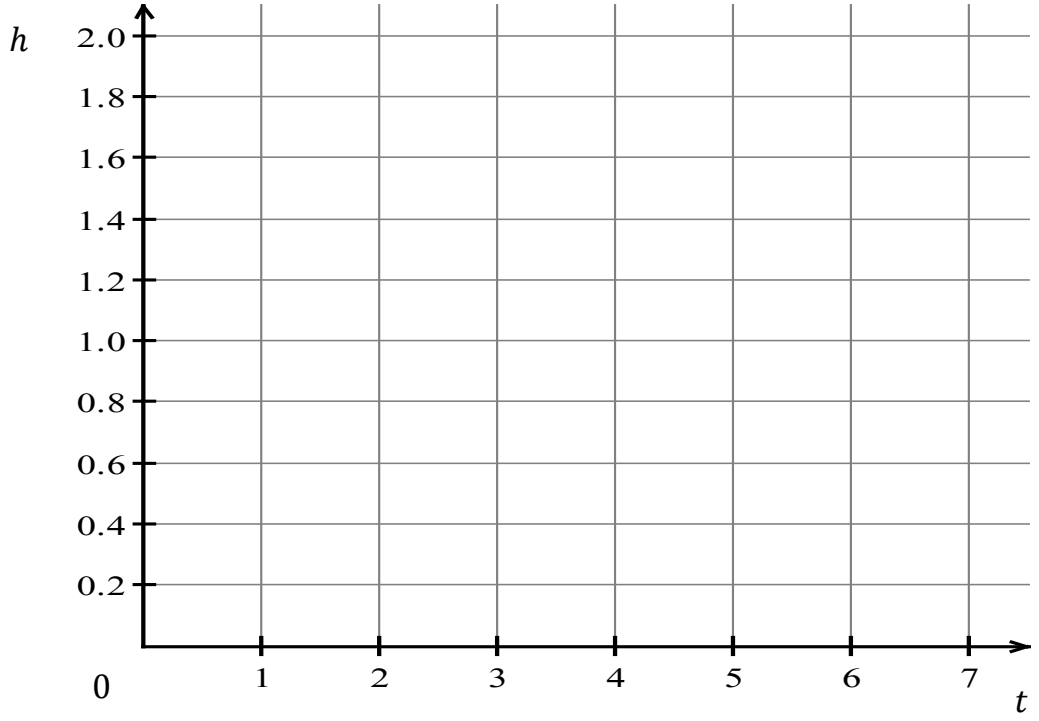
.....

Criterion 5 Total

SPARE DIAGRAM

Section B

Question 24 (a)



SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C Marks = 20.

For Marker Use Only

Question 25

The equation $y = x^3 - 9x^2 + 28x - 28$ is a cubic graph that does not have a stationary point.

(a) What is the value of the slope of the tangent at $x = 3$? (2 marks)

.....

.....

.....

.....

.....

(b) Show the equation has no stationary points. (2 marks)

.....

.....

.....

.....

.....

.....

Section C continues.

For Marker Use Only

Question 26

The price of a new computer can be modelled by the equation $P = 1000e^{-0.5t} + 200$,

where P is the price ($) and t is the time in years.

(a) Calculate the rate of change in price after 5 years. (2 marks)

.....

.....

.....

(b) Calculate the average rate of change over the first 5 years. (2 marks)

.....

.....

.....

.....

.....

.....

Section C continues.

For Marker Use Only

Question 27

Consider the function $f(x) = x^2(x^2 - 2)$.

Find and classify the stationary points of the function. (5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

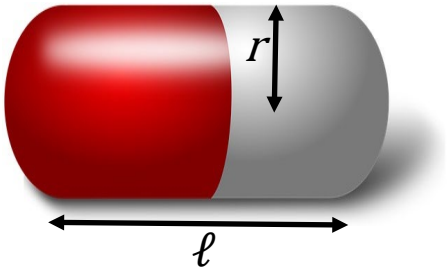
.....

Section C continues.

For Marker Use Only

Question 28

A medicine capsule is constructed by attaching the two halves of a sphere of radius r onto a cylinder with radius r and length ℓ as shown:



(a) The capsule is to have a total surface area (SA) of $7\ \text{cm}^2$.

Hence show that $\ell = -2r + \frac{7}{2\pi r}$. (2 marks)

Useful formulae for surface area:

Sphere: $\text{SA} = 4\pi r^2$ Cylinder: $\text{SA} = 2\pi r\ell$

.....

.....

.....

.....

(b) Using calculus and $V = \frac{4}{3}\pi r^3 + \pi r^2\ell$ find the maximum volume that the capsule can hold with the given surface area. (5 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D Marks = 20.

For Marker Use Only

Question 29

Given $\int_1^4 f(x)dx = 6$ find $\int_1^4 \frac{2f(x)+1}{3}dx$

(3 marks)

.....

.....

.....

.....

Question 30

The velocity of a particle in $\mathrm{m/s}$ is given by $v(t) = 3 + \cos(2\pi t)$ where t is time in seconds.

Find the displacement $s(t)$ in metres after 10 seconds given that $s(0) = 0$. (3 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section D continues.

For Marker Use Only

Question 31

Find the area bounded by $y = e^x - 1$, the x-axis, the line $x = -ln3$ and the line $x = ln3$.

(4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section D continues.

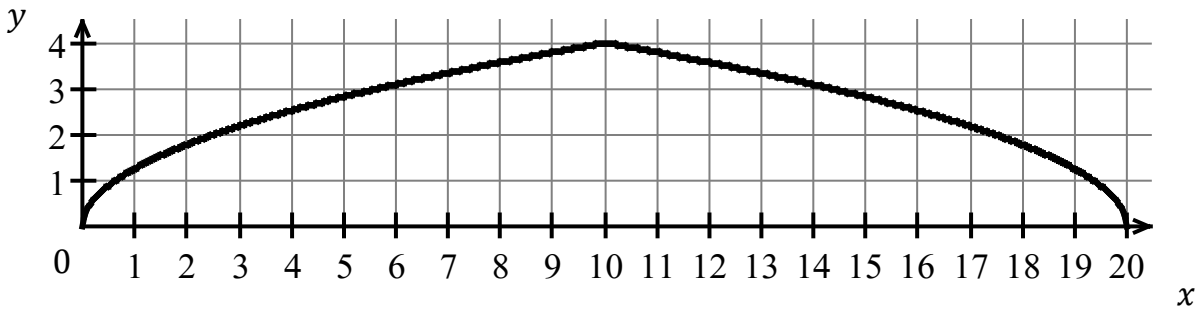
For Marker Use Only

Question 32

Half of the cross-section of a speed hump can be modelled with the equation

$y = 2\sqrt{\frac{2x}{5}}$ for $x \in [0\ ,\ 10]$, where x and y are in centimetres.

The other half from 10 to 20 cm is a reflection of the first function.



The speed hump is to be made of solid concrete.

If there is 16,000 cm^3 of concrete available, how long can the speed hump be made?

Show full algebraic working for the integration required.

(4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

Section D continues.

Section D (continued)

For Marker Use Only

Question 33

The derivative of the function $f(x)$ is given by $f'(x) = x + k\sqrt{x}$, where $k \in \mathbb{R}$.

$f(x)$ has a tangent at $x = 2$ given by $y = 4x - 3$.

Find the function $f(x)$. (6 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Criterion 7 Total

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E Marks = 20.

Question 34

Using standard 6-sided dice, determine the following probabilities:

For Marker Use Only

(a) Rolling a single die and scoring a 6? (1 mark)

.....

(b) Rolling two dice and scoring 5 or more on each one? (1 mark)

.....

.....

(c) Rolling three dice and scoring 4 or more on each one? (1 mark)

.....

.....

(d) What is the most likely scenario of the above three results? (1 mark)

.....

.....

Section E continues.

For Marker Use Only

Question 35

A university study plans to model the length of a species of fish in the ocean.

It is assumed that the length of the fish is normally distributed by $X \sim N(\mu, \sigma^2)$.

Fishers are asked to report the percentage of undersize (less than 30 cm) and very large fish (more than 40 cm) they catch.

They report that 30% are undersize and 5% are very large.

Find the estimates of μ and σ for the population of the species in the ocean. (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 36

A box contains 20 light globes. Each light globe has a 5% chance of being faulty.

(a) What is the probability that at least 3 light globes are faulty in any one box? (2 marks)

.....

.....

.....

.....

(b) A packer opens 10 boxes. What is the probability that fewer than 4 boxes have at least 3 light globes that are faulty? (3 marks)

.....

.....

.....

.....

Section E continues.

Section E (continued)

Question 37

A poll is conducted before an election where Candidate A or B is to be elected President.

The poll says that between 45% and 50% of voters will vote for Candidate A with a confidence interval of 95%.

(a) What is the predicted vote for Candidate A, $\hat{p}$? (1 mark)

.....

.....

(b) How many people were surveyed to give this confidence interval for the poll result? (2 marks)

.....

.....

.....

.....

.....

.....

(c) Candidate A claims that the true proportion in the population that intend to vote for them is p = 51%.

Assuming that the poll is correct, what would the level of confidence have to be if 51% is to be considered within this interval? (4 marks)

.....

.....

.....

.....

.....

.....

.....

.....

For Marker Use Only

Criterion 8 Total

BLANK PAGE