MATHEMATICS METHODS
(MTM415117)

PART 1

Calculators are NOT allowed to be used

Time Allowed: 80 minutes

Candidate Instructions
1. You MUST make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer ALL questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.
5. The 2019 External Examination Information Sheet for Mathematics Methods can be used through the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

Criterion 4 Understand polynomial, hyperbolic, exponential and logarithmic functions
Criterion 5 Understand circular functions
Criterion 6 Use differential calculus in the study of functions
Criterion 7 Use integral calculus in the study of functions
Criterion 8 Understand binomial and normal probability distributions and statistical inference
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Part 1 of the examination is worth 80 marks in total. Each section is worth 16 marks.

You MUST NOT use your calculator(s) during reading time or during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper.

You may start Part 2 during this time, but you cannot use your calculator.

**Part 1 will be collected after 80 minutes (the time allocated to complete Part 1).**

The examination supervisors will advise when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no working is required, markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

For questions worth 2 or more marks you are required to show relevant working.

Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

Spare diagrams are provided at the end of each section for you to use if required. If you use the spare diagrams, you MUST indicate you have done so in your answer to that question.
Answer ALL questions in this section.

This section assesses Criterion 4.

Section A = 16 Marks.

**Question 1**

Show that \((2x+1)\) is a factor of \(2x^3 - 13x^2 + 13x + 10\). (2 marks)

**Question 2**

Determine the equation of the function \(f(x)\), sketched below. (2 marks)

\[
\begin{align*}
\text{Graph showing a curve with a point at (3, 5) and an equation line at } y = 1 \text{.}
\end{align*}
\]
Section A (continued)

Question 3

In the binomial expansion of \((x - 2)^n\) the co-efficient of \(x^{n-1}\) is \(-18\).

Determine the value of \(n\). \hspace{1cm} (2 marks)

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Question 4

Given \(x = e^3\) and \(y = e^4\):

(a) Express \(e^{14}\) in terms of \(x\) and \(y\). \hspace{1cm} (2 marks)

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(b) Find a simplified expression for \(\sqrt{y} + \ln(x)\). \hspace{1cm} (2 marks)

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Section A continues.
Section A (continued)

Question 5

(a) Sketch the function \( y = -\frac{2}{(x + 1)^2} + 3 \), labelling all asymptotes and intercepts on the axes below. (4 marks)

(b) State the range of this function for the restricted domain \( x \in [-2, 1] \). (2 marks)

Criterion 4
Total /16
Question 5
Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B = 16 Marks.

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**Question 6**

(a) Show that \( \frac{11\pi}{15} \) radians is equivalent to \( 132^0 \). (1 mark)

(b) Show that \( \sin \left( \frac{11\pi}{15} \right) = \cos \left( \frac{7\pi}{30} \right) \) (3 marks)

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Section B continues.
Section B (continued)

Question 7

(a) Solve $\sqrt{3} \sin(x) = -\cos(x)$ for $x \in \left[0, 3\pi\right]$.  

(b) Highlight the solutions from part (a) on the graph above.
Question 8
The unit circle below has points evenly spaced around the circumference and overlays a rectangular grid.

Use the rectangular grid and appropriate triangles to estimate values for \( \sin \left( \frac{7\pi}{9} \right) \) and \( \tan \left( \frac{15\pi}{9} \right) \) accurate to two decimal places. (3 marks)

Section B continues.
Section B (continued)

Question 9

Sketch the function \( f(x) = \tan\left(\frac{x - \frac{\pi}{4}}{2}\right) \) where \( x \in \left[0, 2\pi\right] \) on the grid below.

Label key features for \( x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \) and \( 2\pi \). 

(5 marks)
Question 7

Question 8

Question 9
Answer ALL questions in this section.

This section assesses Criterion 6.

Section C = 16 Marks.

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**Question 10**

The function \( f(x) = x^3 - 2x^2 \) has two stationary points.

Find their \( x \) co-ordinates. \( \quad \) \(2 \) marks

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**Question 11**

If \( f(x) = \sin(x) e^{\cos(x)} \) evaluate \( f'(2\pi) \). \(3 \) marks

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Section C continues.
Section C (continued)

Question 12

(a) Show that the derivative of \( g(x) = \log_2(x) \) is \( g'(x) = \frac{1}{x \ln 2} \) by using the Change of Base Theorem.  

(b) The function \( f(x) = \sqrt{x} \) has the same gradient as \( g(x) = \log_2(x) \) for only one \( x \) value. Determine this gradient.

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Section C continues.
Section C (continued)

Question 13

The function \( y = \frac{5}{x^2 + 3} \) is sketched below. The tangents with maximum negative and positive slopes intersect the curve at A and B with a y coordinate of \( \frac{5}{4} \).

Determine the equations of both tangents. (5 marks)

\[
\begin{align*}
\text{For Marker Use Only} & \\
\text{Criterion 6} & \\
\text{Total} & /16
\end{align*}
\]
Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D = 16 Marks.

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**Question 14**

(a) Determine \( \int \left[ (3x + 2)^2 + \sin(2x) + \pi \right] \, dx \). (2 marks)

(b) Show that \( \int_{1}^{4} \frac{1}{2x + 1} \, dx = \frac{\ln 3}{2} \). (2 marks)

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Section D continues.
Section D (continued)

Question 15

Evaluate $\int_{0}^{3} f(x) \, dx$ given $\int_{0}^{3} \left( \frac{1}{10} f(x) - x^2 \right) \, dx = \frac{7}{2}$.

(3 marks)

Question 16

The graph of $f'(x)$ is shown. The curve $y = f(x)$ has a local minimum value of 20.

Find the equation for $f(x)$.

(4 marks)
Section D (continued)

Question 17

The intersection of the four exponential functions and the lines $x = \pm k$ is shown below.

(a) Show that the enclosed area can be written as: $A = 14 \int_{0}^{k} e^{-x} \, dx$.  

(b) Hence, determine a simplified value for $k$ if $A = 12 \text{ units}^2$.  

Criterion 7

Total  

/16
Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E = 16 Marks.

**Question 18**

A discrete random variable has the following probability distribution.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr $(X=x)$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Find the expected value and variance of $X$. (3 marks)

**Question 19**

A binomial random variable has a mean of 16 and a variance of 12. Find both the probability of success, $p$, and the number of trials, $n$. (3 marks)

Section E continues.
Section E (continued)

Question 20

The probability of gaining an apprenticeship after graduating from Vocational College is \( \frac{1}{3} \).

This year a class of 5 students graduated.

(a) Define the distribution and hence determine the probability that at least one student gained an apprenticeship. (3 marks)

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(b) Determine the probability that more than 3 students gained an apprenticeship. (2 marks)

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Section E (continued)

Question 21

Devonport has an average monthly rainfall of 80 mm with a standard deviation of 25 mm. St. Helens has an average monthly rainfall of 65 mm with a standard deviation of 5 mm.

(a) Given that rainfall follows a normal distribution, sketch curves to represent the above data on the axes below. (3 marks)

(b) Explain whether Devonport or St. Helens is more likely to have a monthly rainfall below 50 mm. (2 marks)

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Criterion 8
Total
/16
Question 21

SPARE DIAGRAM

mm rain
MATHEMATICS METHODS
(MTM415117)

PART 2

Calculators are allowed to be used

Time Allowed: 100 minutes

Candidate Instructions

1. You MUST make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer ALL questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 100 minutes in total answering the questions in this booklet.
5. The 2019 External Examination Information Sheet for Mathematics Methods can be used through the examination. No other written material is allowed into the examination.
6. A TASC approved calculator CAN be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

Criterion 4 Understand polynomial, hyperbolic, exponential and logarithmic functions
Criterion 5 Understand circular functions
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Criterion 7 Use integral calculus in the study of functions
Criterion 8 Understand binomial and normal probability distributions and statistical inference

© Copyright for part(s) of this examination may be held by individuals and/or organisations other than the Office of Tasmanian Assessment, Standards and Certification.
Part 2 of the examination is worth 100 marks in total. Each section is worth 20 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification (TASC).

You MUST NOT use your calculator(s) during reading time or during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper.

You may start Part 2 during this time, but you cannot use your calculator.

**Part 1 will be collected after 80 minutes (the time allocated to complete Part 1).**

The examination supervisors will advise when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no working is required, markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

For questions worth 2 or more marks you are required to show relevant working.

Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

Spare diagrams are provided at the end of each section for you to use if required. If you use the spare diagrams, you MUST indicate you have done so in your answer to that question.
Question 22

(a) The function \( f(x) = 2(x - 1)^2 + 2 \) is graphed below.

- Transform \( f(x) \) to \( g(x) \) by reflecting \( f(x) \) in both the \( x \) and \( y \) axes.
- Sketch the resulting graph on the same axes. (1 mark)

(b) State the equation for \( g(x) \).

No working required. (2 marks)
Section A (continued)

Question 23

Determine the cubic function for the sketch below. Give your answer in both factorised and expanded form. (3 marks)

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Question 24

The Richter scale defines the magnitude of an earthquake as, \( M = \log_{10} \left( \frac{I}{S} \right) \) where

\( I \) is the intensity of the earthquake wave, and \( S \) is the intensity of the smallest detectable (or standard) wave.

An earthquake that registered 6.4 in magnitude was followed by another which was 4 times more intense.

Determine the magnitude of the second earthquake accurate to one decimal place. (3 marks)

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Section A continues.
Question 25

Celsius (T), Fahrenheit (F) and Kelvin (K) are temperature scales where:

\[ T(K) = K - 273 \] converts Kelvin to Celsius, and

\[ F(T) = \frac{9}{5} T + 32 \] converts Celsius to Fahrenheit.

(a) Determine a composite function to convert Kelvin to Fahrenheit. (2 marks)

(b) Hence, convert 293 Kelvin to Fahrenheit. (1 mark)

(c) Set up and solve a linear equation to determine the temperature that is the same on both the Celsius and Fahrenheit scales. (2 marks)
Question 26

The function \( f(x) = \frac{-4}{(x-2)^2} + 4 \) is shown on the graph to the left below.

(a) State the largest positive domain of \( f(x) \) so that \( f^{-1}(x) \) exists.

Sketch this inverse on the blank axes above. Include any relevant key points and asymptotes. (3 marks)

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(b) Determine the equation of \( f^{-1}(x) \). (3 marks)

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Question 22

Question 26
Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B = 20 Marks.

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**Question 27**

If \( \sin(\theta) = -\frac{12}{13} \) find exact values for \( \cos(\theta) \) and \( \tan(\theta) \) where \( \theta \in \left[ \pi, \frac{3\pi}{2} \right] \). (3 marks)

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**Question 28**

If \( y = 3\sin\left(\frac{x}{4}\right) \) state the co-ordinates of all maxima where \( x \in [-8\pi, 8\pi] \). (2 marks)

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**Question 29**

Determine the number of solutions for \( \sin(\theta) = -0.7 \) where \( \theta \in [0, 51\pi] \) and provide reasons for your answer. (3 marks)

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Section B continues.
Section B (continued)

Question 30

The musical note, A, can be modelled by the sine function

\[ A(t) = \sin(880\pi t) \]

where \( t \) is measured in seconds.

The sketches below represent 1 period for the function \( A(t) \) and a transformation labelled \( N(t) \).

(a) Determine the period in fractions of a second for \( A(t) = \sin(880\pi t) \).

Hence, determine the number of sine waves per second for the musical note A.

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(b) Noise cancelling headphones remove unwanted sounds by adding functions like the ones shown above. Determine the function \( N(t) \) as a \textbf{horizontal translation} of \( A(t) = \sin(880\pi t) \) shown on the graph above.

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Section B continues.
Section B (continued)

Question 31
A unicycle with a wheel and tyre radius of 25 cm is moving at a constant speed of 500 cm/s. A dot on the tyre tread is at its maximum height at the start.

The height (in cm) of the dot above the ground is modelled against time in seconds.

(a) Given \( C = 2\pi r \) and \( \text{Period} = \frac{C}{\text{Speed}} \), show the period for 1 rotation is \( \frac{\pi}{10} \) seconds.

(1 mark)

(b) Hence, determine a cosine model \( H(t) = a\cos(bt) + c \) for the height of the dot above the ground.

(The cosine sketch below shows 1 rotation of the wheel.)

(3 marks)
Section B (continued)

Question 31 (continued)

(c) Determine the timeframe when \( H(t) > \frac{75}{2} \) cm during the first rotation. (3 marks)

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Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C = 20 Marks.

**Question 32**

The graph below shows a function \( f(x) \).

(a) Sketch a chord on the graph above that represents the average rate of change of \( f(x) \) between \( x = 1 \) and \( x = 16 \). Calculate the average rate of change. (2 marks)

(b) Sketch a line on the graph above that represents the instantaneous rate of change of \( f(x) \) estimated to be the same as the average rate from part (a). (1 mark)
Using first principles, determine the derivative of \( f(x) = (x - 3)^2 \). (3 marks)
Question 34

For the function below:

- The gradient increases as $x \to -\infty$
- Stationary points occur at $x = A$ and $C$
- The gradient is steepest at $B$ when $A < x < C$
- The gradient is steepest at $D$ when $x > C$
- The gradient approaches zero as $x \to \infty$

Sketch a possible derivative function on the axes below. (4 marks)
Question 35

The point \((x, y)\) lies on the function \(y = \sqrt{x}\), at a distance \(L\) from the point \((3, 0)\).

(a) Use the distance formula \(L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\) to show that \(L = \sqrt{x^2 - 5x + 9}\). (1 mark)

(b) Hence, use calculus to determine the minimum distance. (Show all working, but justification of the minimum is not required). (4 marks)
Section C (continued)

Question 36

Use calculus and the quotient rule to find and classify the stationary points of the function

\[ f(x) = \frac{(x+1)^2}{x}. \]  

(5 marks)

For Marker Use Only

Criterion 6
Total

/20
Question 32

Question 34
Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D = 20 Marks.

**Question 37**

(a) Write a definite integral to determine the shaded area of the function $f(x)$ below.  
(2 marks)

(b) Explain whether the sign of $\int_a^b f(x)\,dx$ is likely to be positive or negative.  
(1 mark)

**Question 38**

(a) Differentiate $x \cos(bx)$ where $b$ is a constant.  
(1 mark)

(b) Hence, determine $\int x \sin(bx)\,dx$ using integration by recognition.  
(3 marks)
Section D (continued)

Question 39

Find the area enclosed by $f(x) = \sqrt{x-1}$, the line $y=2$ and the $x$ and $y$ axes. (5 marks)
Section D (continued)

Question 40

The vertical motion of a rocket shortly after launch has an acceleration
\[ a = 1.8e^{0.03t} \] where \( a \) is measured in \( \text{m/s}^2 \).

When the velocity was first measured it was 2 m/s and the rocket was
displaced 100m above the ground.
\( (t = 0 \) is defined as the time when the first measurement was taken).

(a) Determine equations for both velocity and displacement. (6 marks)

(b) Hence, calculate the velocity and displacement when the fuel tanks were jettisoned
109 seconds after the first measurement. (2 marks)
Answer **ALL** questions in this section.

This section assesses **Criterion 8.**

Section E = 20 Marks.

**Question 41**

A survey of 400 college students found that 45% travel to college by bus.

(a) Estimate the standard deviation of the sample distribution of the college students travelling by bus. Give your answer to 4 decimal places.  

(b) Determine the $z$ score for a 90% confidence interval to 4 decimal places.  

(c) Hence, determine the 90% confidence interval for the proportion of students travelling by bus to college.  

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**Section E continues.**
Question 42

A discrete random variable $X$ relates to a 4 sided “biased” die with the following distribution in terms of the constant $k$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Pr($X = x$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$6k^3 - \frac{1}{5}$</td>
</tr>
<tr>
<td>2</td>
<td>$5k^2 - 2k^3$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{4}{5} - 2k$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{2}{5} - k^3$</td>
</tr>
</tbody>
</table>

(a) Show the only valid solution for $k$ is $\frac{1}{3}$ and explain why any other solutions are rejected. (You may use your calculator to solve equations). (4 marks)
Section E (continued)

Question 42 (continued)

The distribution for \( \Pr(X = x) \) is given below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1 ( \quad )</th>
<th>2 ( \quad )</th>
<th>3 ( \quad )</th>
<th>4 ( \quad )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pr(X = x) )</td>
<td>( \frac{1}{45} )</td>
<td>( \frac{13}{27} )</td>
<td>( \frac{2}{15} )</td>
<td>( \frac{49}{135} )</td>
</tr>
</tbody>
</table>

James and Hollie rolled the biased die once to play a game. They each start with 12 tokens.
- If 1, 2 or 3 is the outcome, James gains one token from Hollie.
- If 4 is the outcome, James loses 2 tokens to Hollie.

(b) Calculate the expected value of the game.
Determine the number of games expected before one player has all of the tokens.
(4 marks)
Section E (continued)

Question 43

A breakfast food company sells muesli in boxes labelled 650 grams. The weight of muesli in the boxes is normally distributed with a standard deviation of 9 grams.

(a) The company requires 55% of boxes to be over the labelled weight (650 g).

Determine the mean weight required, to the nearest 0.1g. (4 marks)

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(b) A box of muesli is deemed to be underweight if its weight is more than 5% below the labelled 650g.

Calculate the probability that a box of muesli is underweight. Hence, determine the number of underweight boxes in a run of 20 000 boxes. (4 marks)

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