

GENERAL MATHEMATICS

COURSE CODE: MTG315120

BIVARIATE DATA ANALYSIS

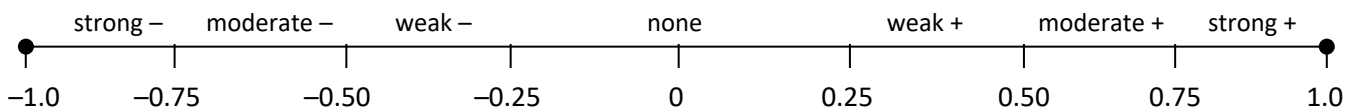
Linear Functions

$$y = ax + b$$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = a(x - x_1)$$

Correlation coefficient (r)



Coefficient of determination (r^2) - % of the variation in 'y' that can be associated with the variation in 'x'

Residuals Residual = actual data value – modelled value

Time Series Analysis

Seasonally adjusted data

1. Find average for each cycle
2. Divide actual data by cycle averages
3. Calculate seasonal indices
4. Deseasonalise the data:

$$\text{deseasonalised data} = \frac{\text{actual data}}{\text{seasonal index}}$$

GROWTH & DECAY IN SEQUENCES

Arithmetic Sequences

$$a, a + d, a + 2d, \dots, a + (n-1)d$$

$$\text{Where } t_n = a + (n-1)d$$

Arithmetic Series

$$a + (a + d) + (a + 2d) + \dots + (a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}(2a + (n-1)d)$$

Geometric Sequences

$$a, ar, ar^2, \dots, ar^{n-2}, ar^{n-1}$$

$$\text{Where } t_n = ar^{n-1}$$

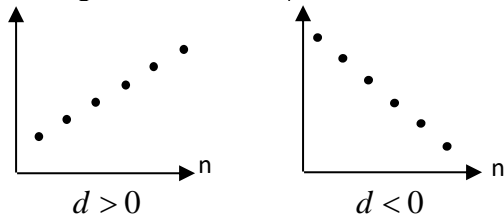
Geometric Series

$$a + ar + ar^2 + \dots + ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

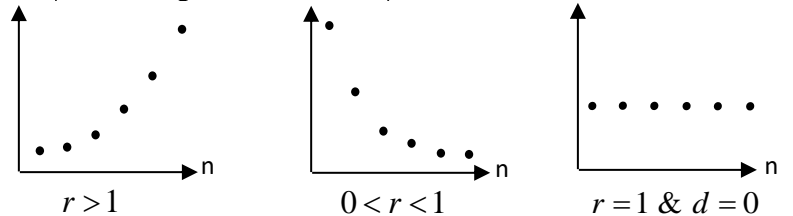
Arithmetic Sequence Graphs

Linear growth and decay



Geometric Sequence Graphs

Exponential growth and decay



Steady State

First order difference (recurrence) equations

$$t_{n+1} = rt_n + d, \text{ where } t_1 \text{ or } t_0 \text{ is given} \quad \text{or } t_{n+1} = at_n + b$$

$$\text{Annuities in arrears: } t_{n+1} = rt_n - d \quad t_0 = a \quad \text{Annuities in advance: } t_{n+1} = r(t_n + d) \quad t_0 = 0$$

FINANCE

Simple interest:

$$I = PRT$$

Compound interest

$$A = P(1+i)^n$$

Straight line depreciation:

$$V = -Dn + C$$

Reducing Balance depreciation:

$$A = P(1-i)^n$$

Effective interest:

$$E = (1+i)^n - 1$$

Annuities in advance/Sinking funds:

$$F = \frac{R(1+i)[(1+i)^n - 1]}{i} \quad \text{OR} \quad t_{n+1} = r(t_n + d) \quad t_0 = 0$$

Annuities in arrears/Reducible balance:

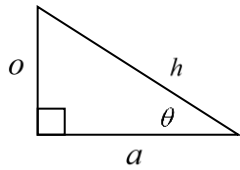
$$P = \frac{R[1 - (1+i)^{-n}]}{i} \quad \text{OR} \quad t_{n+1} = rt_n - d \quad t_0 = a$$

Perpetuities:

$$P = \frac{R}{i}$$

TRIGONOMETRY

Right-Angle Trigonometry



$$\sin \theta = \frac{o}{h} \quad \cos \theta = \frac{a}{h} \quad \tan \theta = \frac{o}{a}$$

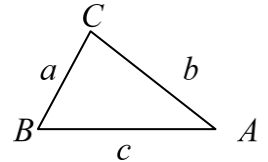
Non-Right-Angle Trigonometry

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



Area of a triangle

$$\text{Area} = \frac{1}{2} ab \sin C$$

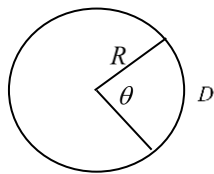
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$

Bearings - 'true' 215°T or 'reduced' S35°W

Earth Geometry

Radius of the Earth = 6 371 km

Arc length



kilometres

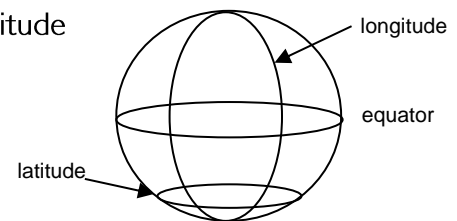
Great circle

$$D = \frac{2\pi R\theta}{360}$$

Small circle

$$D = \frac{2\pi R\theta \cos \alpha}{360}$$

Latitude & Longitude



nautical miles (n miles)

$$D = 60\theta$$

$$D = 60\theta \cos \alpha$$

where α = latitude

Angular Separation on Great Circles

$$\cos \theta = \sin(\text{latP}) \cdot \sin(\text{latQ}) + \cos(\text{latP}) \cdot \cos(\text{latQ}) \cdot \cos(\text{longitudinal difference})$$

Standard Time Zones

$$= \text{UTC} \pm \frac{\text{longitude } ^\circ\text{E} / ^\circ\text{W}}{15} \text{ hours (round to nearest hour)}$$

Australian Time Zones

WST = UTC + 8 hours (Western Standard Time)

CST = UTC + 9.5 hours (Central Standard Time)

EST = UTC + 10 hours (Eastern Standard Time)

Estimated Time of Arrival (ETA)

ETA = depart time + travel time \pm standard time difference

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

GRAPHS & NETWORKS

Euler's formula

$$V + F - E = 2$$

Critical path analysis

Critical path: longest path from start to finish

Earliest start time (EST)

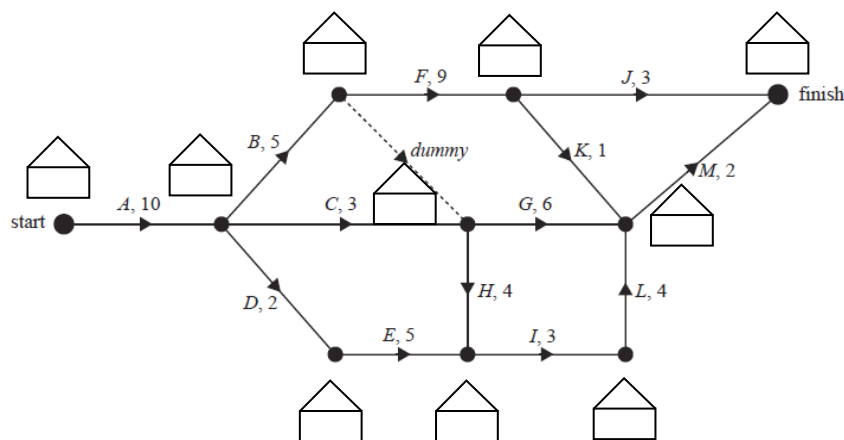
Latest finish time (LFT)

Latest start time (LST): = LFT – activity duration

Float time = time available – activity duration

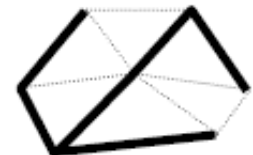
= (LFT – activity duration) – EST

= LST – EST



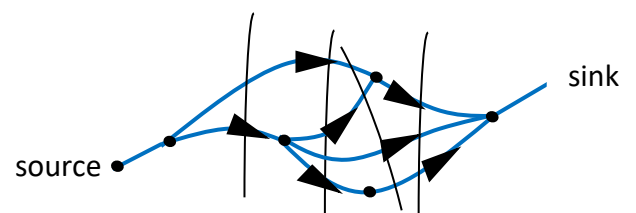
Spanning Trees

Prim's Algorithm: Start with any vertex then to shortest edge.....



Network Flow

'minimum cut' = 'maximum flow'



Hungarian Algorithm

1. Row reduction?
2. Column reduction?
3. Hungarian algorithm
 - Select **smallest uncovered number**.
 - Add that number to numbers that are crossed twice.
 - Subtract that number from any uncovered number.
4. Assignment