EXTERNAL ASSESSMENT INFORMATION SHEET

SEQUENCES AND SERIES

Arithmetic sequences	$u_n = a + (n-1)\,d$ $S_n = \frac{n}{2}(2a + (n-1)\,d) = \frac{n}{2}(a + \ell)$
Geometric Sequences	$u_n = a\,r^{n-1}$ $S_n = \frac{a(1-r^n)}{1-r}$ if $r \neq 1$ or na when $r = 1$ $S_\infty = \frac{a}{1-r}$ if $\lvert r \rvert < 1$
	$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$
Convergent sequence	The sequence $\{a_n\}$ converges to a finite limit L if, for any $\varepsilon > 0,\ \exists\, N(\varepsilon)$ such that $\lvert a_n - L \rvert < \varepsilon\ \forall\, n > N$
Divergent sequence	The sequence $\{a_n\}$ diverges to positive infinity if, for any $\kappa > 0,\ \exists\, N(\kappa)$ such that $a_n > \kappa\ \forall\, n > N$ The sequence $\{a_n\}$ diverges to negative infinity if, for any $\kappa > 0,\ \exists\, N(\kappa)$ such that $a_n < -\kappa\ \forall\, n > N$
MacLaurin's Series for $f(x)$	$f(x) = f(0) + f'(0) \times x + f''(0) \times \frac{x^2}{2!} + f'''(0) \times \frac{x^3}{3!} + \cdots + f^{(n)}(0) \times \frac{x^n}{n!} + \ldots$

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MATRICES AND LINEAR TRANSFORMATIONS

Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = \lvert A \rvert = ad - bc$
Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, ad \neq bc$
Dilation matrices	$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}$
Shear matrices	$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$
Rotation matrix	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
Reflection matrix	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$
Equation of a circle centre (h, k) and radius r	$(x-h)^2 + (y-k)^2 = r^2$
Equation of an ellipse centre (h, k) horizontal semi-axis of length a vertical semi-axis of length b	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Equation of a plane	$ax + by + cz = d$
Equation of a line	$(x_0 + at, y_0 + bt, z_0 + ct), t \in \mathbb{R}$ (parametric form) $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$, $a, b, c \neq 0$ (symmetric form)

COMPLEX NUMBERS

For $z \in \mathbb{C}$	$z = a + ib = r(\cos\theta + i\sin\theta) = r\, cis\theta = r\, e^{i\theta}$ $Re(z) = a$ and $Im(z) = b$ $\lvert z \rvert = \sqrt{a^2 + b^2} = r$
De Moivre's Theorem	$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) = r^n cis\, n\theta = r^n e^{in\theta}$

TRIGONOMETRY

Identities	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$
Pythagorean identities	$\cos^2\theta + \sin^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
Double angle identities	$\sin 2A = 2\sin A\cos A$ $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$ $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
Compound angle identities	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
	$2\sin A\cos B = \sin(A+B) + \sin(A-B)$ $2\cos A\sin B = \sin(A+B) - \sin(A-B)$ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$ $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$ $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$ $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$ $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$
Exact Values	(see table below)
Conversion	To convert from radians to degrees multiply by $\dfrac{180}{\pi}$ To convert from degrees to radians multiply by $\dfrac{\pi}{180}$

Exact Values

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$undefined$	0	$undefined$	0

CALCULUS

Definition of Derivative:

$$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

Differentiation Formulae

Function	Derivative
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x \quad or \quad \frac{1}{\cos^2 x}$
e^x	e^x
$\log_e x \quad or \quad \ln x$	$\frac{1}{x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
a^x	$a^x \ln a$
$\log_a x$	$\frac{1}{x \ln a}$

Integration Formulae

Function	Integral
a	$ax + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$(ax+b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln \lvert x \rvert + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\arcsin \frac{x}{a} + c$ or $-\arccos \frac{x}{a} + c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \arctan \frac{x}{a} + c$
a^x	$\frac{a^x}{\ln a} + c$
$\log_a x$	$\frac{x \ln \lvert x \rvert - x}{\ln a} + c$

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Differentiation Rules

	Function	Rule	Function	Rule
Product Rule	$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$	$u \cdot v$	$v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$
Quotient Rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\{g(x)\}^2}$	$\frac{u}{v}$	$\frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$
Chain Rule	$g\{f(x)\}$	$g'\{f(x)\} \cdot f'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du} \cdot \frac{du}{dx}$

Integration by Parts Formula:

$$\int f(x) \cdot g'(x)\ dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x)\ dx + c$$

Area under curve:

$$A = \int_a^b y\,dx \quad \text{or} \quad A = \int_a^b x\,dy$$

Volume of revolution:

$$V = \pi \int_a^b y^2\,dx \quad \text{or} \quad V = \pi \int_a^b x^2\,dy$$

ALGEBRAIC FORMULAE

General Quadratic Formula:

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Indices	Logarithms
$a^x \times a^y = a^{x+y}$ $a^x \div a^y = a^{x-y}$ $(a^x)^y = a^{x \times y}$ $(a)^{\frac{1}{y}} = \sqrt[y]{a}$ $(a)^{\frac{x}{y}} = \sqrt[y]{a^x}$	$y = a^x \Leftrightarrow x = \log_a y \quad a > 0, y > 0$ $\log_a x + \log_a y = \log_a xy$ $\log_a x - \log_a y = \log_a \frac{x}{y}$ $\log_a x^n = n \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_a 1 = 0$ $\log_a a = 1$

Binomial Expansion:

$n! = n(n-1)(n-2)(n-3) \ldots \times\ 3 \times 2 \times 1$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$(x+y)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots \binom{n}{n-1} x\, y^{n-1} + \binom{n}{n} y^n$

Absolute Value:

$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$