Tasmanian Certificate of Education

GENERAL MATHEMATICS

Senior Secondary

Subject Code: MTG315115

External Assessment

2015

Part 1 – Bivariate Data Analysis

Time: approximately 36 minutes

On the basis of your performance in this examination, the examiners will provide results on the following criterion taken from the course statement:

Criterion 4 Demonstrate knowledge and understanding of bivariate data analysis.

<table>
<thead>
<tr>
<th>Section Total</th>
<th>/36</th>
</tr>
</thead>
</table>

Pages: 12
Questions: 3

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CANDIDATE INSTRUCTIONS

You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.

The 2015 External Examination Information Sheet for General Mathematics can be used throughout the examination.

No other written material is allowed into the examination.

**ALL** questions in this part should be attempted.

Answers must be written in the spaces provided on the examination paper.

This examination is 3 hours in length. In total it is recommended that you spend approximately 36 minutes answering the questions in this part.

Graph paper is provided in the booklet when required.

Logical and mathematical presentation of answers and the statement of the arguments leading to your answer will be considered when assessing this part.

You are expected to provide a calculator approved by the Office of Tasmanian Assessment, Standards and Certification.

All written responses must be in English.

For questions worth 1 or 2 marks, working does **not need** to be shown.

For questions worth 3 or more marks, you are **required to show** relevant working.

**Spare diagrams have been provided in the back of the booklet for you to use if required.**

**If you use either of these spare diagrams you MUST indicate you have done so in your answer to that question.**
Question 1 (Approximately 6 minutes)

A group of students and teachers was surveyed as to the type of smart phone they owned.

Of the 32 teachers interviewed, 19 owned an iPhone, whilst 40 students owned an iPhone and 50 students owned an android phone.

(a) Use this information to complete the two-way table below.  

<table>
<thead>
<tr>
<th></th>
<th>Students</th>
<th>Teachers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPhone</td>
<td>40</td>
<td>19</td>
<td>59</td>
</tr>
<tr>
<td>Android phone</td>
<td>50</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>32</td>
<td>82</td>
</tr>
</tbody>
</table>

(b) (i) Complete the table below to present this data in percentage terms and complete the segmented column graph below to display this information.  

<table>
<thead>
<tr>
<th></th>
<th>Students %</th>
<th>Teachers %</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPhone</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Android phone</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Briefly summarise your findings.  

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**Question 2** (Approximately 17 minutes)

Leonie measured the depth of a car tyre tread (T), in mm, from when the tyre was new, until it had been driven on for a distance (D) of sixty thousand km. Her results are shown below.

<table>
<thead>
<tr>
<th>Distance driven D (thousand km)</th>
<th>Tyre tread T (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.0</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
</tr>
<tr>
<td>10</td>
<td>4.9</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
</tr>
<tr>
<td>30</td>
<td>3.2</td>
</tr>
<tr>
<td>40</td>
<td>2.3</td>
</tr>
<tr>
<td>50</td>
<td>2.0</td>
</tr>
<tr>
<td>60</td>
<td>1.8</td>
</tr>
</tbody>
</table>

(a) Which is the dependent (response) variable?  (1 mark)

(b) State the linear equation for the line of best fit for this data. Give your answer to three decimal places.  (2 marks)

(c) State the correlation coefficient for the linear equation in part (b). Explain what this means in terms of the modelled data.  (2 marks)

(d) Use algebra and your equation from part (b) to predict the distance driven when the tyre tread is 4.4 mm.  (3 marks)

Question 2 continues.
Question 2 (continued)

(e) Comment on the reliability, or otherwise, of your answer in part (d). (2 marks)

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(f) Prepare a scaled residual plot for the linear model in part (b) on the grid below. (3 marks)

![Residual Plot Grid]

(g) State whether or not the linear equation you have found in (b) is a good model for the data or not. Give reasons for your choice. (4 marks)

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Question 3 (Approximately 13 minutes)

The quarterly water usage, Q1, Q2, Q3 and Q4 in kilolitres (kL), of a household over the years 2012 to 2014 is shown below. (Where Q1 is quarter 1, etc.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Quarterly average (kL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>195</td>
<td>95</td>
<td>53</td>
<td>65</td>
<td>102.0</td>
</tr>
<tr>
<td>2013</td>
<td>150</td>
<td>85</td>
<td>47</td>
<td>35</td>
<td>79.2</td>
</tr>
<tr>
<td>2014</td>
<td>140</td>
<td>99</td>
<td>37</td>
<td>25</td>
<td>75.2</td>
</tr>
</tbody>
</table>

(a) Include the Q1 to Q4 raw data for 2014 on the graph opposite. (1 mark)

(b) Use the quarterly average figures above to complete the Q4 column below and, hence, determine the seasonal index for Q4. Give your answers to three decimal places. (2 marks)

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Seasonal Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>1.912</td>
<td>0.931</td>
<td>0.520</td>
<td>1.888</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>1.893</td>
<td>1.073</td>
<td>0.593</td>
<td>1.107</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>1.860</td>
<td>1.316</td>
<td>0.492</td>
<td>0.535</td>
<td></td>
</tr>
</tbody>
</table>

(c) Deseasonalise the data for 2014 and include these points on the table below. Plot the deseasonalised data on the graph opposite. (2 marks)

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>103</td>
<td>86</td>
<td>99</td>
<td>138</td>
</tr>
<tr>
<td>2013</td>
<td>79</td>
<td>77</td>
<td>88</td>
<td>74</td>
</tr>
<tr>
<td>2014</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Question 3 continues.
Question 3 (continued)

(d) Explain what has been the effect of deseasonalising the water usage data and describe the long term (secular) trend of the water usage over the three years. (3 marks)

(e) From the information for the three years, 2012 to 2014, which quarter had the highest water usage on a deseasonalised basis? Explain what this means. (2 marks)

Question 3 continues.
Question 3 (continued)

The equation of the linear trend line of the deseasonalised data was found to be:

\[ \text{Water usage} = -3.822x + 110.750, \] where \( x \) is the quarter number

(f) Use this equation and another calculation to predict the actual water usage for this family in the first quarter of the year 2016. (3 marks)
**SPARE DIAGRAMS**

**Question 1 (b) (i)**

[Diagram showing a bar chart with 100% on the y-axis and D (thousand km) on the x-axis.]

**Question 2 (f)**

[Diagram showing a residual graph with D (thousand km) on the x-axis and Residuals on the y-axis.]
SPARE DIAGRAMS

Question 3

![Graph showing water usage (kL) over quarters](image)

- **Raw data**
- **Deseasonalised data**
Tasmanian Certificate of Education

GENERAL MATHEMATICS

Senior Secondary

Subject Code: MTG315115

External Assessment

2015

Part 2 – Growth and Decay in Sequences

Time: approximately 36 minutes

On the basis of your performance in this examination, the examiners will provide results on the following criterion taken from the course statement:

**Criterion 5** Demonstrate knowledge and understanding of growth and decay in sequences.

<table>
<thead>
<tr>
<th>Section Total</th>
<th>/36</th>
</tr>
</thead>
</table>

Pages: 12
Questions: 4

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No other written material is allowed into the examination.

**ALL** questions in this part should be attempted.

Answers must be written in the spaces provided on the examination paper.

This examination is 3 hours in length. In total it is recommended that you spend approximately 36 minutes answering the questions in this part.

Graph paper is provided in the booklet when required.

Logical and mathematical presentation of answers and the statement of the arguments leading to your answer will be considered when assessing this part.

You are expected to provide a calculator approved by the Office of Tasmanian Assessment, Standards and Certification.

All written responses must be in English.

For questions worth 1 or 2 marks, working does **not need** to be shown.

For questions worth 3 or more marks, you are **required to show** relevant working.

A spare diagram has been provided in the back of the booklet for you to use if required.

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Question 4 (Approximately 8 minutes)

Fatima starts a new job where her first year salary is $54,000. Each year after that her salary increases by $2,450.

(a) How much does Fatima earn in her second year in this job? (1 mark)

(b) Write an arithmetic sequence rule for her salary. (2 marks)

(c) Algebraically, determine the year in which Fatima’s salary first reaches $71,150. (3 marks)

(d) Determine the total amount of money Fatima earns from when she starts her job to the end of the year she earns $71,150. (2 marks)
**Question 5** (Approximately 7 minutes)

The graph opposite shows how the number of bacteria in a culture increases over a period of five hours.

(a) What feature of this graph shows that the number of bacteria increases exponentially? (1 mark)

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(b) Using the two points indicated on the graph above, algebraically determine the rule for the geometric sequence representing the number of bacteria over this time period. (Give the common ratio to three decimal places.). (3 marks)

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Question 5 continues.
Question 5 (continued)

(c) At the end of which hour will the number of bacteria first exceed 100,000? Comment on the reliability of your prediction. (3 marks)
Question 6 (Approximately 9 minutes)

A hardware shop sells 40 lawn mowers in one week and then 46 lawn mowers the following week.

(a) If an **arithmetic sequence** model applies to their sales, determine how many lawn mowers will be sold in the fifth week. (2 marks)

(b) If a **geometric sequence** model applies to their sales, determine how many lawn mowers will be sold in the fifth week. (3 marks)

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Question 6 continues.
Question 6 (continued)

(c) If the shop actually sold 364 lawn mowers in total from weeks 1 to 6, which of the sequence models in (a) and (b) seems to best model the lawn mower sales? Explain.

(4 marks)

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Question 7 (Approximately 12 minutes)

A farmer estimates that he has 2 500 wallabies on his farm. The wallaby population increases by 20% each year, whilst the farmer removes 200 wallabies each year.

The first order difference (recurrence) equation that gives the number of wallabies on the farm at the end of each year (n = number of years) can be written as:

\[ W_{n+1} = 1.20 \ W_n - 200, \] where \( W_0 = 2 \ 500 \) wallabies

(a) Use this equation to predict the number of wallabies at the end of the first and second years. (2 marks)

End year 1: ........................................................................................................................................
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End year 2: ........................................................................................................................................
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(b) Prepare, on your calculator, a graph which shows the wallaby population for \( n = 0 \) to 5 years. Copy this graph onto the axes below. (3 marks)
Question 7 (continued)

(c) If, instead, the farmer removes 800 wallabies each year, write the new first order difference (recurrence) equation below.

Graph these figures on the graph in part (b) for \( n = 0 \) to 5 years.

Describe how the wallaby numbers change now.

(d) Algebraically, determine how many wallabies the farmer would have to remove each year if the population was to remain at (a constant) 2 500 at the end of every year.
SPARE DIAGRAM

Question 7

Number of wallabies

year
On the basis of your performance in this examination, the examiners will provide results on the following criterion taken from the course statement:

**Criterion 6** Demonstrate knowledge and understanding of standard financial models.
CANDIDATE INSTRUCTIONS

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No other written material is allowed into the examination.

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**Question 8** (Approximately 5 minutes)

At the beginning of March, Sophie had $3 000 in her bank account.

On 25 March Sophie withdrew $1 500 and on 2 May she deposited $2 000.

(a) If the account pays interest of 3.65% p.a. on a daily balance, how much interest will she have earned at the end of 30 June? (4 marks)

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(b) What will be the balance of Sophie’s account at the end of 30 June? (1 mark)

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**Question 9** (Approximately 8 minutes)

Aisha wants to have $17,500 in a bank account after 2 years so that she can buy a new car.

(a) **Algebraically**, calculate the amount of money she needs to invest to have $17,500 if interest is 4.40% p.a., compounded monthly. 

(b) Aisha finds out that her $17,500 new car depreciates at rate of 20% p.a. on a **reducing balance** basis. Determine what her car will be worth after 5 years of ownership.

(c) Aisha sells her car after 5 years for the figure in part (b) and puts this money towards another new car.

If the price of a comparable new car increased at a rate of 1.3% p.a. over the same time period, determine how much extra Aisha would need in order to buy such a car in 5 years time.
Question 10 (Approximately 10 minutes)

A company deposits $2 000 each quarter into an annuities account which earns 4.80 % p.a., compounded monthly.

(a) Determine the balance of this annuities account after 5 years.  

(b) After five years, the company invests $45 000 of the money earned from part (a) into another account that pays an interest rate of 5.0% interest compounding annually.

If the interest earned each year is awarded as an annual scholarship, determine the value of the annual scholarship.

Question 10 continues.
Question 10 (continued)

(c) If the annual scholarship was instead $5 000 each year, determine how long this could be awarded for. (4 marks)

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**Question 11** (Approximately 13 minutes)

Bill owes $11 000 on his credit card and he is charged an interest rate of 19.00 % p.a., compounded monthly, on this debt.

If Bill was to repay this credit card debt at the repayment figure of $183.14 per month, he would pay the debt after 16 years exactly.

(a) Determine how much interest Bill will pay on his credit card debt of $11 000 over the 16 years. (2 marks)

(b) Using an appropriate formula, show that Bill owes $10 741 (to the nearest dollar) after 2 years of repayments. (3 marks)

(c) After two years, Bill is given the option of paying off this $10 741 at an interest rate of 1.99% p.a., compounded monthly. He first has to pay a transfer fee of 1.25% on the figure from part (b).

Including the transfer fee of 1.25%, if Bill takes up this option, how much will he owe now? (2 marks)

Question 11 continues.
Question 11 (continued)

(d) Bill decides to repay $500 every month towards paying off the amount owing in part (c) at the 1.99% p.a. compounded monthly interest rate.

Determine how much time and how much money Bill will save by repaying the debt this way in comparison with the original payment terms given at the beginning of the question. (6 marks)

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Time saved: ..................................................................................................................................
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Money saved: ..................................................................................................................................
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On the basis of your performance in this examination, the examiners will provide results on the following criterion taken from the course statement:

**Criterion 7** Demonstrate knowledge and understanding of applications of trigonometry.
CANDIDATE INSTRUCTIONS

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**Question 12** (Approximately 6 minutes)

A cargo ship sails **due east** from Christchurch (44.0˚S, 172.7˚E) to the Chatham Islands (44.0˚S, 176.0˚E).

(a) What distance, in nautical miles, does the cargo ship sail? (2 marks)

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(b) If the cargo ship left Christchurch at 7:10 pm on 2 May and travelled at an average speed of 16.0 knots, determine the time it arrived in the Chatham Islands. (Note that both Christchurch and the Chatham Islands are in the same time zone). (4 marks)

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**Question 13** (Approximately 5 minutes)

A dark shade cloth ‘sail’ has the dimensions shown below.

![Diagram of the sail](image)

(a) Determine the area of the smaller triangle above with side lengths of 5.0, 3.0 and 7.1 m. Give your answer to two decimal places. (3 marks)

(b) Determine the area of the dark shade cloth ‘sail’. Give your answer to two decimal places. (2 marks)
**Question 14** (Approximately 14 minutes)

Gary and Kathryn flew from Buenos Aires (35°S, 58°W) to Melbourne (38°S, 145°E) via the **shortest possible route**.

They departed Buenos Aires at 11:00 am on Wednesday, 25 July. The flight path their plane took is shown on the map below.

(a) Determine the **standard time difference** between Melbourne and Buenos Aires.  

(2 marks)

(b) If Kathryn made a phone call from Buenos Aires at 9:45 am on Wednesday, 25 July to a friend in Melbourne, at what Melbourne (EST) time did her friend receive the phone call?  

(2 marks)

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**Question 14 continues.**
Question 14 (continued)

(c) Briefly explain why the flight path shown in the diagram opposite appears to be very curved. (2 marks)

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(d) Determine the distance (in km) that the plane flew from Buenos Aires to Melbourne. (4 marks)

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(e) If their plane arrived in Melbourne at 3:27 pm on Thursday, 26 July, determine the average speed of the plane, in km per hour. (4 marks)

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**Question 15** (Approximately 11 minutes)

Andrew (A), Brenda (B) and Chris (C) are standing on a level field. Andrew (A) notices that Brenda (B) is 77 metres away from him at a bearing of 064° T, whilst Chris (C) is 42 m away from him at a bearing of 140° T as shown in the diagram below.

(a) Complete the diagram above with the information given in the question and hence show that the angle at A (angle CAB) is 76°.  
(3 marks)

(b) Determine the **distance** from Brenda to Chris. Give your answer to one decimal place.  
(2 marks)

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**Question 15 continues.**
Question 15 (continued)

(c) Determine the angle at B (angle ABC) and hence determine the true bearing from Brenda to Chris. Give your answers to the nearest minute. (6 marks)

Angle at B: ......................................................................................................................

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True bearing from Brenda to Chris:

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SPARE DIAGRAM

Question 15

A

B

C
Tasmanian Certificate of Education

GENERAL MATHEMATICS

Senior Secondary
Subject Code: MTG315115

External Assessment

2015

Part 5 – Graphs and Networks

Time: approximately 36 minutes

On the basis of your performance in this examination, the examiners will provide results on the following criterion taken from the course statement:

**Criterion 8**  Demonstrate knowledge and understanding of graphs and networks.

<table>
<thead>
<tr>
<th>Section Total</th>
<th>/36</th>
</tr>
</thead>
</table>

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This part is worth 36 marks.

Graph paper is provided in the booklet when required.

Logical and mathematical presentation of answers and the statement of the arguments leading to your answer will be considered when assessing this part.

You are expected to provide a calculator approved by the Office of Tasmanian Assessment, Standards and Certification.

All written responses must be in English.

For questions worth 1 or 2 marks, working does **not need** to be shown.

For questions worth 3 or more marks, you are **required to show** relevant working.

Spare diagrams have been provided in the back of the booklet for you to use if required. If you use either of these spare diagrams you **MUST** indicate you have done so in your answer to that question.
**Question 16** (Approximately 7 minutes)

The network below represents the water flow, in litres per minute, in a series of pipes.

(a) Identify the **source** and the **sink** of this network. (1 mark)

Source: ........................................................................................................................................

Sink: ...........................................................................................................................................

(b) Give the values of the cuts I, II, III, IV and V in the spaces provided in the above network. Hence, determine the **maximum flow** of the network. (3 marks)

Maximum flow: ..........................................................................................................................

(c) In order to increase the water flow in this system, it is planned to upgrade the pipe **section CF**. What is the maximum worthwhile upgrade to this pipe section and what would the new maximum flow rate then be? Explain. (3 marks)

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Question 17 (Approximately 11 minutes)

The network below shows the roads, in km, that exist between nine farms, A to I.

(a) Describe a Hamiltonian Circuit that starts and finishes at farm A. What distance does this circuit cover? (2 marks)

...................................................................................................................................................

...................................................................................................................................................

Distance covered: ...........................................................................................................................

...................................................................................................................................................

(b) A delivery truck travels from farm A to farm I. Using an algorithmic approach on the network diagram above, determine the shortest route the truck can take and the distance that it travels. (4 marks)

Shortest route from A to I: ............................................................................................................

...................................................................................................................................................

Distance covered: ...........................................................................................................................

Question 17 continues.
Question 17 (continued)

(c) An electricity company needs to upgrade its connections between these farms. Sketch on the diagram below, the minimum spanning tree and hence determine the minimum distance covered.

Minimum distance covered: .................................................................
..............................................................................................................
..............................................................................................................

(d) Hire cars can only travel on sealed roads in Tasmania. The reduced network diagram below shows only sealed roads between the farms.

State an Eulerian path where a hire car can travel along each and every road exactly once.

..............................................................................................................
..............................................................................................................
**Question 18** (Approximately 9 minutes)

The **bipartite graph** below, shows the favourite subjects of four students, Anna, Bertrand, Cam and David.

(a) David lists his favourite subjects as General Maths and English Communications. **Include** this information on the bipartite graph. (1 mark)

\[
\begin{align*}
\text{Anna} & \quad \text{General Maths} \\
\text{Bertrand} & \quad \text{Physical Sciences} \\
\text{Cam} & \quad \text{English Communications} \\
\text{David} & \quad \text{Psychology}
\end{align*}
\]

(b) Which is a favourite subject of both Bertrand and Cam? (1 mark)

\[
\begin{align*}
\text{Physics} \\
\text{English}
\end{align*}
\]

As part of a General Maths activity, Anna, Bertrand, Cam and David participate in a competition that has four different activities, E, F, G and H.

The time, in minutes, that each student took to complete each activity is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>12</td>
<td>13</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Bertrand</td>
<td>10</td>
<td>8</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Cam</td>
<td>7</td>
<td>9</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>David</td>
<td>8</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

**Question 18 continues.**
Question 18 (continued)

In order to best assign these students to an activity it is necessary to perform a row reduction then a column reduction.

(c) Complete the **row reduction** process in the table below.  

\[
\begin{array}{cccc}
E & F & G & H \\
\hline
\text{Anna} & & & \\
\text{Bertrand} & & & \\
\text{Cam} & & & \\
\text{David} & & & \\
\end{array}
\]

Explain why it is **not possible** to complete the allocation of activities after this step.

........................................................................................................................................

........................................................................................................................................

(d) Complete the **column reduction** below and then **allocate** each student to a task.  

\[
\begin{array}{cccc}
E & F & G & H \\
\hline
\text{Anna} & & & \\
\text{Bertrand} & & & \\
\text{Cam} & & & \\
\text{David} & & & \\
\end{array}
\]

Allocation: ..........................................................................................................................

........................................................................................................................................

Total time taken: .................................................................................................................

........................................................................................................................................
**Question 19** (Approximately 9 minutes)

The times taken, in days, for the tasks A to L to be completed are shown in the project network below. Note that some earliest start times (EST) are shown.

(a) Determine the **earliest starting times** (EST) for activities F and G.  
(2 marks)

(b) Complete all of the (time keeping) boxes in the project network above.

Use this information to determine the critical path and indicate the minimum time to complete this project.  
(5 marks)

**Critical path:**

**Minimum time to complete the project:**

---

**Question 19 continues.**
Question 19 (continued)

(c) Determine by how long the following two activities could be delayed before the minimum time to complete this network was affected. (2 marks)

Activity H: ........................................................................................................................................
........................................................................................................................................
Activity K: ........................................................................................................................................
........................................................................................................................................
SPARE DIAGRAMS

Question 17 (b)

Question 17 (c)
SPARE DIAGRAMS

Question 17 (d)

A B C D E F G H I

Question 19

0 10 B, 14 D, 15 F, 9 E, 21 I, 19 K, 9 L, 4

Start A, 10 C, 12 G, 10 H, 7

Finish
General Mathematics

Subject Code: MTG315115

2015 External Examination Information Sheet

BIVARIATE DATA ANALYSIS

Linear Functions

\[ y = ax + b \]

\[ a = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ y_1 - y = a(x - x_1) \]

Correlation coefficient \((r)\)

<table>
<thead>
<tr>
<th>strong –</th>
<th>moderate –</th>
<th>weak –</th>
<th>none</th>
<th>weak +</th>
<th>moderate +</th>
<th>strong +</th>
</tr>
</thead>
<tbody>
<tr>
<td>–1.0</td>
<td>–0.75</td>
<td>–0.50</td>
<td>–0.25</td>
<td>0</td>
<td>0.25</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Coefficient of determination \((r^2)\) - % in the variation in ‘y’ that can be associated with the variation in ‘x’

Residuals  Residual = actual data value – modelled value

TIME SERIES ANALYSIS

Seasonally adjusted data

1. Find average for each cycle
2. Divide initial data by cycle averages
3. Calculate seasonal indices
4. Deseasonalise the data:

\[ \text{Deseasonalised data} = \frac{\text{actual data}}{\text{seasonal index}} \]
GROWTH & DECAY IN SEQUENCES

Arithmetic Sequences
\[ a, a + d, a + 2d, \ldots, a + (n - 1)d \]
Arithmetic Series
\[ a + (a + d) + (a + 2d) + \ldots + (a + (n - 1)d) \]
\[ S_n = \frac{n}{2} (a + l) = \frac{n}{2} (2a + (n-1)d) \]

Geometric Sequences
\[ a, ar, ar^2, \ldots, ar^{n-1} \]
Geometric Series
\[ a + ar + ar^2 + \ldots + ar^{n-1} \]
\[ S_n = \frac{a(1 - r^n)}{1 - r} \]

Arithmetic Sequence Graphs
Linear growth and decay

Geometric Sequence Graphs
Exponential growth and decay

Steady State

First order difference (recurrence) equations
\[ t_{n+1} = rt_n + d, \quad \text{where } t_1 \text{ or } t_0 \text{ is given} \]
or
\[ t_{n+1} = at_n + b \]

Annuities in arrears: \[ t_{n+1} = rt_n - d \quad t_0 = a \]
Annuities in advance: \[ t_{n+1} = r(t_n + d) \quad t_0 = 0 \]

FINANCE

Simple Interest:
\[ I = PRT \]
where \( T = \frac{\text{days}}{365} \)

Compound interest
\[ A = P(1+i)^n \]

Straight line depreciation:
\[ V = -Dn + C \]

Reducing Balance depreciation:
\[ A = P(1-i)^n \]

Effective interest:
\[ E = (1+i)^n - 1 \]

Annuities in advance/Sinking funds:
\[ F = \frac{R(1+i)[(1+i)^n-1]}{i} \quad \text{OR} \quad t_{n+1} = r(t_n + d) \quad t_0 = 0 \]

Annuities in arrears/Reducible balance:
\[ P = \frac{R[1-(1+i)^n]}{i} \quad \text{OR} \quad t_{n+1} = rt_n - d \quad t_0 = a \]

Perpetuities:
\[ P = \frac{R}{i} \]
APPLIED GEOMETRY

Right Angle Trigonometry

\[ \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \]

Non Right Angle Trigonometry

**Sine Rule**
\[ \frac{a}{\sin A} = \frac{b}{\sin B} \]

**Cosine Rule**
\[ a^2 = b^2 + c^2 - 2bc \cos A \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} \]

Area of a triangle
\[ \text{Area} = \frac{1}{2} ab \sin C \quad \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a + b + c}{2} \]

Bearings – ‘true’ 215°T or ‘reduced’ S 35°W

Earth Geometry

Radius of the earth = 6371 km

**Arc length**
\[ D = \frac{2\pi R \theta}{360} \]

**Longitude & Latitude**
\[ D = 60\theta \]

**Great circle**
\[ D = \frac{2\pi R \theta \cos \alpha}{360} \]

**Small circle**
\[ D = \frac{2\pi R \theta}{360} \]

Angular Separation on Great Circles:
\[ \cos \theta = \sin (\text{lat P}) \sin (\text{lat Q}) + \cos (\text{lat P}) \cos (\text{lat Q}) \cos (\text{longitudinal difference}) \]

Standard Time Zones
\[ = \text{UTC} \pm \frac{\text{longitude \degree E/W}}{15} \text{ hours} \text{ (round to nearest hour)} \]

Estimated Time of Arrival (ETA)
\[ \text{ETA} = \text{depart time} + \text{travel time} \pm \text{standard time difference} \]

Australian Time Zones
- WST = UTC + 8 hours (Western Standard Time)
- CST = UTC + 9.5 hours (Central Standard Time)
- EST = UTC + 10 hours (Eastern Standard Time)

Estimated Speed = \( \frac{\text{distance}}{\text{time}} \)
**GRAPHS & NETWORKS**

**Euler’s formula**
\[ V + F - E = 2 \]

**Critical path analysis**

**Critical path**: longest path from start to finish

**Earliest starting time (EST)**

**Latest finish time (LFT)**

**Latest start time (LST)**: \( LFT - \text{activity duration} \)

**Float time** = time available – activity duration
\[ = (LFT - \text{activity duration}) - \text{EST} \]
\[ = \text{LST} - \text{EST} \]

**Spanning Trees**

**Prim’s Algorithm**: Start with any vertex then to shortest edge…..

**Network Flow**

‘minimum cut’ = ‘maximum flow’

**Hungarian Algorithm**

1. Row reduction?
2. Column reduction?
3. Hungarian algorithm
   - Select smallest uncovered number.
   - Add that number to numbers that are crossed twice.
   - Subtract that number from any uncovered number
4. Assignment