General Mathematics
Course Code: MTG315115

Part 1 - Bivariate Data

Most students appeared to have enough time to complete the section.

Most students appeared be reasonably prepared for the examination of this section.

Examiners found that the main areas that students and teachers need to address are in developing skills to relate their linear equation knowledge to the question at hand. Many had a basic understanding of a linear relationship, but had trouble applying it further. Students need to be encouraged to use their reading time to read the questions thoroughly and decipher what is being asked.

General communication needs to be improved, with the expected inclusion of units with all answers, use of appropriate number of decimal places and scaling and labelling of axis.

Student’s interpretive and analysis skills need practice. They should be encouraged to answer the worded questions by using values in their explanations and be clear in justifying choices and selections. Also students should be encouraged to make concise worded answers, and to make sure to check they have actually answered the question asked of them. If they are asked to refer to previous answers such as part (d) and (e) they should make sure to illustrate this within their answer.

Students must also provide some Mathematical 'substance' in questions worth 3 or more marks. Students are reminded that rulers are a necessary part of their stationary equipment in this subject and appropriate use of a ruler is encouraged.

Spare graphs are provided at the end of the paper and students need to be encouraged that it is more appropriate for them to use these when errors have occurred rather than crossing out and writing over the top.

Question 1

(a) This question was quite well done by most students. If students made an error with this question they confused cyclic with random or seasonal.

(b) This question was quite well done by most students. However students need to be encouraged to always show their working illustrating how they obtained their answers.

Question 2

(a) This question was quite well done by most students. Rounding was the biggest issue. Plus graphing accuracy and skills need attention. Rulers need to be encouraged to assist with this.

(b) This question was quite poorly done by most students. Common errors included using the two point formula incorrectly and instead of using the calculator, starting their first list on the value 1 rather than 2, incorrect rounding and incorrect number of decimal places, and obvious data entry errors in their calculator.

(c) This question was reasonably well done by most students. Students should be encouraged to provide an equation in the previous part (b) of the question to then demonstrate their ability to use it to provide an answer for this question. Many students knew to substitute 10 into the equation (although some did 9) but then did not round to the nearest whole number of children.
(d) This question was quite well done by most students. To get full marks for this question students were required to determine the actual month and not leave the answer in decimal form.

(e) This question was quite poorly done by most students. Most students did not recognise that the answer to part (c) was based on extrapolation and therefore unreliable even though it was close to the dataset. Some students did not read the question properly and did not determine reliability for both part (c) and (d).

Question 3

(a) This question was very poorly done by most students. Most students did not understand the concept of gradient or y-intercept. They did not refer to the values and were not able to interpret these values to the variables Temperature and Cold Drinks Sold. They also did not recognised the significance of the negative number for the y-intercept and comment upon its relevance.

(b) This question was quite well done by most students. Some students confused the terminology weak positive correlation with strong relationship. Some students also incorrectly referred to the Temperature as the causation for the Cold Drinks Sold, and others did not use the appropriate variables and instead referred generally to x and y.

(c) This question was quite poorly done by most students. Most students were able to answer part i) but many had trouble explaining the effect on the linear model part ii). Most referred to the residual graph and were able to recognise the point as an outlier, but were not able to explain the effect on the linear model.

(d) This question was quite well done by most students. Some students confused C and T or used x and y, or had trouble rounding appropriately.

(e) This question was quite well done by most students. Most students plotted the graph quite well, but a few missed some information off the graph, such as a label or a scale. A few students joined the points as well.

(f) This question was quite well done by most students. Most students said the linear equation modelled the data well. Some students did not refer to the r value (part d) or give enough information in support of the residual graph (part e). Some students wanted an exactly even number of points either side of the x-axis, which is not necessary.

Part 2 - Growth and Decay in Sequences

Most students appeared to have enough time to complete this section.

Most students appeared to be poorly prepared for the examination section.

Examiners found that the main areas that students and teachers need to address are the reading and interpretation of the questions asked. It is important to answer the question asked. General communication needs to be improved, with the expected inclusion of units with all answers and use of appropriate number of decimal places. They should also consider the reasonableness of their answers and make appropriate corrections. Many students either ignored the instructions to show solutions algebraically or were not able to demonstrate this skill.

As marks are used in the exam it is essential to show clear working to obtain full marks in any question worth 3 or more marks. This is essential in any “show that” type questions where students need to treat these questions in the same way as any question where the answer is not given, a substitution type approach is not acceptable and minimal marks are given in this case.

Students are encouraged to become more familiar with their calculator in sequence mode. Students appeared to be using main/solve more frequently than the sequences part on their calculator often making it much harder to answer the question correctly. They need to pay more attention to starting values in tables and to be able to
generate tables effectively. They also need to be able to increase values to obtain a clear idea of what is happening to the sequence over a much longer period of time.

Student’s answers to the worded questions need to include enough written comments. Also students should be encouraged to make concise worded answers. It should be noted that for three and four mark questions students should give three of four valid and separate points for full marks. For full marks detailed answers displaying understanding was required and not just facts or calculations.

Graphing skills also need addressing. Students often found scaling a challenge. Discrete points are required on the graph and some students mistakenly want to put in trend lines.

A general lack of understanding of recursive equations needs to be addressed as many students gave incomplete equations by not giving the initial value.

**Question 4**

(a) This question was quite well done by most students. However some students did not find the correct term but the value for 2012 instead. Some students gave an incorrect value for difference.

(b) This question was quite well done by most students. Some students missed the plus between the a and \((n-1)\) and sometimes had a negative instead of positive value for 2000.

(c) This question was quite well done by most students. Students’ algebraic skills need to be improved as some algebraic manipulation was often omitted. Students who used their calculator to solve where not awarded full marks as an algebraic solution was required. Some students found the correct value for \(n\) but did not give the correct year as 2016 was required rather than year 9 as this what was used in the table.

(d) This question was very poorly done by most students. Many students did not recognise the \(r\) value was 1. Many students did not include the \(T_0/T_1\) term in their solution.

(e) This question was quite well done by most students. Often the Units (tonnes) was not given but no units or ‘number of cherries’ was used instead. Some students chose the incorrect number for \(n\) (did not take into account ‘inclusive’).

**Question 5**

(a) This question was very well done by most students.

(b) This question was quite poorly done by most students. Students often found incorrect value for \(r\) in Jasmin’s salary. Many students considered the 10th year only and more was required for full marks. In most cases not enough information or evidence was given. Not enough written comments. Interpretation of question was poor.

**Question 6**

(a) This question was quite well done by most students. However some students were making substitutions for \(R_n\) and some students need to work on their rounding skills.

(b) This question was very poorly done by most students. Many students did not read this question carefully as both the running and cycling data was required with many students only plotting the data for either running or cycling but not both. Some students did not label axes or chose a poor scale. Extra care in plotting points is also required as many students were not careful enough. Many students joined the points which is not appropriate for discrete data in sequences and series.

(c) This question was quite well partly answered by most students. Most students found max running distances but did not give an appropriate time for when this occurred. The description of trend was very poor with students often describing it as linear or exponential which is incorrect.
(d) This question was quite well done by most students. Some students got incorrect value for k but still wrote the difference equation. Many students got the correct value for k but did not fully answer the question as no difference equation was given.

**Question 7**

(a) This question was quite poorly done by most students. Many students do not understand that in any “show that” type questions students need to treat these questions in the same way as any question where the answer is not given, a substitution type approach is not acceptable and minimal marks are given in this case. Some students found r but did not clearly find the value of a.

(b) This question was quite poorly done by most students. Many students did not subtract the r value from 1 to give the correct percentage. Some students gave their answer as a decimal rather than a percentage.

(c) This question was very well done by most students. Some students wrote ‘during the 18th year’ rather than at the ‘end of the 19th year’. Again demonstrating the difficulty students find in interpreting the n values. Some students attempted to solve the equation equal to zero rather than approaching it from a table in sequence mode.

(d) This question was quite well done by most students. However many students did not understand that \( r = 1 - 0.18 \).

(e) This question was quite well done by most students. Many students got the correct value but did not describe the change in enough detail.

(f) This question was quite well done by most students. Many students did not show their calculation for r and did not receive full marks for the question. Although many students made errors in calculating r most were able to place r in the difference equation and give the corresponding percentage decrease.

**Part 3 - Finance**

Most students appeared to have enough time to complete this section.

Most students appeared to be quite well prepared for the examination section.

All students need to read the questions with care and fully answer the questions asked. It is essential they spend time reading the whole question carefully before diving in. They should also consider the reasonableness of their answers and make appropriate corrections. Students should ensure they have appropriately answered the question. This may require them to go back once they have ‘finished’ the question and check their answer. Many marks were missed this way in this section.

General communication needs to be improved, with the expected inclusion of units with all answers and use of appropriate number of decimal places. In Finance, students should leave their final answer as whole dollars or to the closest cent. Rounding appropriately was a major issue. Students should be careful of carrying over rounding errors and in Finance all significant figures should be used until the final answer. Students are encouraged to leave \( i \) in all the formulas as a fraction rather than converting to a decimal and rounding.

Examiners found one of the main areas that students and teachers need to address is the development of an understanding of real life examples in Finance, and therefore the extracting the relevant information from the question and what strategies are required to solve these problems. The students’ lack of understanding of the simple interest problems was an indication of this.

While many questions did not require an algebraic solution, calculator methods MUST be accompanied by detailed specifications of the inputs used. This includes students writing out either (begin) or (end) in every annuities question.
As marks are now used in the exam it is essential to show clear working to obtain full marks in any question worth 3 or more marks. This is essential in any “show that” type questions where students need to treat these questions in the same way as any question where the answer is not given, a substitution type approach is not acceptable and minimal marks are given in this case.

It is strongly suggested that the effective rate is only used as a comparison rate and when any other type of problem is asked the nominal rate is used.

**Question 8**

(a) This question was quite poorly done by most students. Many students did not take the easier option of doing the conversions in the Finance mode of the calculator. A significant number of students showed that they did not know how to use the effective interest formula. Particularly how to input the interest rate, most common errors were n times the number of years or forgetting to divide the rate by the number of periods. Some students did not fully answer the question by writing which Bank was the best option.

(b) This question was quiet well done by most students. Either nominal or effective rate could be used but the relevant adjustments to ‘i’ and ‘n’ were not adjusted. Many students used the effective interest rate but still multiplied the number of periods by the number of years. There were also a significant number who used bank A, or made a comparison between Bank A and B and ignored the investment altogether.

(c) This question was quite poorly done by most students. Recurrence relations are a part of finance and students need to be prepared for this. Many students used the annuities in advance formula from the sheet instead of just one for a geometric series. There were also considerable errors with changing 4.4% to 1.044, either leaving the 1 off or getting the decimal place wrong. Many students did not include the T₀ value as part of their answer.

**Question 9**

(a) This question was quite poorly done by most students. Many students did not notice the change of months. Students need to be careful calculating dates and keep an eye out for different months. Students should make use of their calculator to check the date they calculate if they do this by hand. Also make sure the time ‘t’ in days is divided by the 365 days in the year. I is the interest only, many students thought this was the principle plus the interest. Recognise that ‘r’ is a decimal and that to convert to a percentage they must times by 100. Some students were put off by r being 125% and thinking this value too large. Many students crossed or rubbed out the correct answer before reattempting to try and make the answer more comfortable indicating very little understanding of this type of loan and that the rate can be greater than 100%. There were also many common answers based on wrong days.

(b) This question was quiet well done by most students. There were usual problems with some students not dividing the rate by 100 to convert to a decimal and the days by 365 to convert to years. Some students did not fully answer the question by finding the amount saved.

**Question 10**

(a) This question was very well done by most students. Students must note (begin) or (end) in conveying their Finance mode calculator details.

(b) This question was extremely well done.

(c) This question was quiet well done by most students. Many students did not know how to incorporate the $3500 into their calculations. A significant number thought that this value would ‘kick-start’ the saving and used it as the present value at the beginning of Sophie’s saving instead of the correct solution of only having to save up to $21500. Some students did not fully answer the question by finding the extra amount!
Question 11

(a) This question was very well done by most students. In order to ensure full marks (although not penalised), students should make sure that they specify their answer to the nearest cent. Again (begin) or (end) must be specified. Students should know that they need to solve for the answer $222\text{,}1$, rather than substitute the value into the formula.

(b) This question was quiet well done by most students. Some students found the total paid over 5 months or 300 months rather than 60 months. Some students did not know how to answer this type of question and tried using annuity formulas.

(c) This question was quite poorly done by most students. Few students recognised that this question required 2 parts. Few students were able to correctly calculate the amount owing with 20 years remaining. A large number of students answered this question by only working out the time with a new interest rate. Students incorrectly subtracted the result of part b) from the original value, or simply calculated how long it would take to pay back the full $32\text{,}000\text{,}000 with a different interest rate of 5.2%.

(d) This question was quiet well done by most students. For full marks students needed to use partial payments rather than whole payments. The amount saved on the loan needs to be calculated from how much has been paid, or utilising the difference in the number of payments made, rather than how much there is left on the loan.

Part 4 - Trigonometry

Most students appeared to have enough time to complete this section.

Most students appeared to be well prepared for the examination section.

Examiners found that the main areas that students and teachers need to address is in the area of communication, the general reading of the question and construction of diagrams. All students need to read the questions with care and answer the questions asked. They should also consider the reasonableness of their answers and make appropriate corrections. General communication needs to be improved, with the expected inclusion of units with all answers and the use of the appropriate number of decimal places. Students need to be careful to not round within a question round for the final answer only.

As marks are now used in the exam it is essential to show clear working to obtain full marks in any question worth 3 or more marks. This is essential in any “show that” type questions where students need to treat these questions in the same way as any question where the answer is not given, a substitution type approach is not acceptable and minimal marks are given in this case.

The calculator should be checked that it is in degree mode as some students had correct working but incorrect answers. Care is required when using degrees, minutes and seconds function on the calculator when converting decimal hours to minutes.

Diagrams need to be legible and clearly show the angles and lengths. A significant number of students tried to use right angle triangle rules for non-right angled triangles.

Students are advised to take the time to understand what the question is asking before commencing.

There was quite a split between a large number of students who did very well and a large group who did very little or nothing at all. Some students clearly had a preference for either world geometry or trigonometry and did far better in one half than the other.

Overall we were quite pleased with how the students did.
Question 12

(a) This question was very well done by most students. However some students did not use the cosine rule formulae, and many neglected to show the square root.

(b) This question was well done by most students. Clear working was not always shown and marks were deducted if only answers were given. Some students had issues with calculator errors when using Heron’s rule. Some students calculated the same area twice. Some students did not use correct units of m² for the area.

Question 13

(a) This question was very well done by most students. Nearly all students drew the diagram correctly however the A and B positions were not always given.

(b) This question was reasonably well done by most students. Many students had difficulty finding the correct angle at the top of the non-right angled triangle. Many students did not identify this as a two part problem with a non-right angle triangle problem first and then a second step to solve for height using a right-angled triangle. Some students failed to include appropriate units.

Question 14

(a) This question was very well done by most students.

(b) This question was very well done by most students. Some students substituted incorrectly, some tried to (incorrectly) use the sine rule. Many did not show the inverse cos step.

(c) This question was quite poorly done by most students. More than half the students did not even attempt to calculate the bearing and many only found the distance between the two boats. Some students found the angle but few students then managed to find the bearing correctly. Students were unable to select the correct angles to be used in order to calculate the correct bearing angle giving instead the reverse of the bearing asked for.

Question 15

(a) This question was quiet well done by most students. Most students did not clearly state that Fremantle was ahead of Colombo.

(b) This question was reasonably well done by most students. Some students subtracted the zone time difference instead of adding.

(c) This question was well done by most students. Most students knew to use the angular separation formula first, however, there were a number of students who found the longitude difference incorrectly or used longitude instead of latitude in the formula. Some students could not find the angle correctly. Most students used the correct formula to find the distance in nautical miles but some students are still confused when to use which formula.

(d) This question was reasonably well done by most students. Most students knew how to do this question but students were not able to clearly communicate a solution and there were many errors in finding the correct arrival time. Incorrect adjustment for zone time difference was common. Often a time was just given with no clear indication of how it was found. Very few students actually obtained the correct arrival time and date. Converting from decimal time (148.43 hours) to days, hours and minutes proved difficult. Students confused hours and minutes with decimal hours giving the incorrect arrival time as they tried to add decimals with minutes. There was also some confusion between how to write hours and minutes and some students were writing time as degrees and seconds.
This question was quite poorly done by most students. This was the only question that was commonly left blank. Some students used the great circle formula rather than small circle formula. Some students who found the correct longitudinal difference did not correctly find the position coordinates.

Part 5 - Graphs and Networks

Most students appeared to have enough time to complete this section.

Most students appeared to be reasonably well prepared for the examination section.

Examiners found that the main areas that students and teachers need to address is to ensure that all students have read the questions with care, looking for strategic words to assist them to answer the questions asked. Students need to make sure that they use the formula sheet and communicate clearly the steps that they have used to obtain an answer. Students should also think about the validity of an answer, and check for errors if it appears unreasonable. There were a lot of processes that students needed to be careful in their execution of and many lost marks for careless errors. Generally most students gave units where required but some did miss marks by numerical or unit errors.

Spare graphs are provided at the end of the paper and students need to be encouraged that it is more appropriate for them to use these when errors have occurred rather than crossing out and writing over the top.

Question 16

(a) This question was very well done by most students.

(b) This question was reasonably well done by most students. Many students did not include enough detail or could not articulate how the flow can be restricted because not all pipes have the same capacity. Some students just restated the question.

(c) This question was quite well done by most students. Students should make sure to include an adequate number of cuts (as many as possible and a minimum of four). Students need to take care with adding the pipes in a cut and to include the units of KL per minute correctly.

(d) This question was quite poorly done by most students. Students could identify the pipes to upgrade but had trouble quantifying or illustrating the concept of increasing the capacity up to the next minimum cut. A lot of students missed giving two possible solutions and only gave one. Many forgot to include units as part of their answer.

Question 17

(a) This question was very well done by most students.

(b) This question was very well done by most students. Basically if they knew what an Eularian path was they were able to answer the question.

(c) This question was very poorly done by most students. Most students did not get the correct path. Lots were confused as to what to do and tried to include all the points. Students could not visualise and look ahead towards the shortest path. Adding up was poorly done and many did not write what numbers they had added which could have awarded them part marks. Some paths were clearly quite large and students did not consider the reasonableness of their answer, illustrating poor estimation skills. Very few used the algorithmic approach to assist them.

(d) This question was quite well done by most students. Most students understood the concept of a minimum spanning tree but made some errors including missing the vertex k, or not choosing 1 or 2 shorter lines. Students would be advised to check after they have completed the minimum spanning tree that there are no edges that they could exchange for shorter ones. Some students made errors in their arithmetic.
Question 18

(a) This question was quite well done by most students. Many students were able to complete the project network graph. Some students did not use the starting diagram to complete the network graph, others left the project “I” adrift rather than connecting it to the finish. Some forgot to create a vertex between G and J and others forgot to include arrows as well.

(b) This question was reasonably well done by most students. Students had trouble explaining the concept as it applied to this diagram. They tended to regurgitate a learnt response and found it difficult to demonstrate their understanding. They needed to refer to activities as preceding or being completed prior to starting another activity.

(c) This question was reasonably well done by most students. Time keeper boxes were inconsistently completed and had numerical errors. Many did not get the box between activity A and D because of the dummy edge. Some students were able to get the path from the table. Units of days was well done.

(d) This question was quite poorly done by most students. A lot did not explain or show their working well. For a 3 mark question working is part of the required answer. Successful responses calculated the float and then explained the concept of float to include that the activity could be delayed for up to and including 23 days. Some students found the float and then incorrectly took the activity length off. Others did not use the formula sheet to find the float and so were not clear as to what they were doing.

Question 19

(a) This question was reasonably well done by most students. Many students had rehearsed the question but not well. Many knew where to start but got confused as they went through the column reduction. Many ignored the numbers they had crossed out in the column reduction. Some students forgot to indicate they had accomplished the assignment or completed the test in order to reach a conclusion. Some chose smallest numbers rather than the zeros.

(b) This question quite well done by most students. Some students were careless in writing the answer clearly. Most were able to do an assignment even if they could not complete part a) correctly.

(c) This question was quite well done by most students. Some students made arithmetic errors and did not multiply by 8 to get the total cost but showed their working to be able to pick up part marks.
GENERAL MATHEMATICS  
(MTG315115)

PART 1 – Bivariate Data Analysis

Time: 36 minutes

Candidate Instructions

1. You MUST make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.

2. Answer ALL questions. Answers must be written in the spaces provided on the examination paper.

3. You should make sure you answer all parts within each question so that the criterion can be assessed.

4. This examination is 3 hours in length. It is recommended that you spend approximately 36 minutes in total answering the questions in this booklet.

5. The 2016 External Examination Information Sheet for General Mathematics can be used throughout the examination. No other written material is allowed into the examination.

6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Demonstrate knowledge and understanding of bivariate data analysis.

Section Total: /36

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Additional Instructions for Candidates

Logical and mathematical presentation of answers and the statement of the arguments leading to your answer will be considered when assessing this part.

You are expected to provide a calculator approved by the Office of Tasmanian Assessment, Standards and Certification.

For questions worth 1 mark, whilst no workings are required, markers may consider appropriate step(s) taken to come to an answer.

For questions worth 2 or more marks, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 3 or more marks, you are **required to show** relevant working.

**Spare diagrams have been provided in the back of the booklet for you to use if required.**

If you use either of these spare diagrams you **MUST** indicate you have done so in your answer to that question.
Question 1 (approximately 5 minutes)

(a) Using two of the terms shown below, describe the trend and the secular (long term) trend indicated in the two graphs below: (2 marks)

**trend terms**: seasonal, cyclic, or random,

**secular trend terms**: upwards, a downwards or no long-term trends.

![Graph 1: Share Price ($) vs. Months](image)

![Graph 2: Retail Sales vs. Quarters](image)

(b) The quarterly electricity usage of a household, in kilowatt hours (kWh), and some seasonal index and deseasonalised quarterly usage figures related to these are shown in the table below. (3 marks)

<table>
<thead>
<tr>
<th>Quarter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity usage (kWh)</td>
<td>220</td>
<td>421</td>
<td>400</td>
<td>147</td>
</tr>
<tr>
<td>Seasonal index</td>
<td>0.60</td>
<td>1.45</td>
<td>1.27</td>
<td>0.68</td>
</tr>
<tr>
<td>Deseasonalised Electricity Usage (kWh)</td>
<td>367</td>
<td>290</td>
<td>315</td>
<td>216</td>
</tr>
</tbody>
</table>

(i) Determine the seasonal index for quarter 2.

\[ 4 - (0.60 + 1.27 + 0.68) = 1.45 \]

(ii) Deseasonalise the electricity usage for quarter 2 and hence determine which quarter used the least amount of electricity in seasonally adjusted terms.

\[ \frac{421}{1.45} = 290.345 \approx 290 \text{ kWh for quarter 2 (deseasonalised)} \]

Quarter 4 used the least amount of electricity in seasonally adjusted terms.
Question 2 (approximately 14 minutes)

A new child care centre opened in January. The number of children attending on the first day of each month is shown in the table below.

(a) (i) Calculate the missing 3-point moving average figures and include these in the table below. (Give your numbers to one decimal place.) (2 marks)

<table>
<thead>
<tr>
<th>Month</th>
<th>Month number</th>
<th>Children</th>
<th>3-pt moving average</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>2</td>
<td>46</td>
<td>54.3</td>
</tr>
<tr>
<td>March</td>
<td>3</td>
<td>65</td>
<td>56.0</td>
</tr>
<tr>
<td>April</td>
<td>4</td>
<td>57</td>
<td>57.3</td>
</tr>
<tr>
<td>May</td>
<td>5</td>
<td>50</td>
<td>59.3</td>
</tr>
<tr>
<td>June</td>
<td>6</td>
<td>71</td>
<td>62.0</td>
</tr>
<tr>
<td>July</td>
<td>7</td>
<td>65</td>
<td>70.3</td>
</tr>
<tr>
<td>August</td>
<td>8</td>
<td>75</td>
<td>74.3</td>
</tr>
<tr>
<td>September</td>
<td>9</td>
<td>83</td>
<td>70.7</td>
</tr>
<tr>
<td>October</td>
<td>10</td>
<td>54</td>
<td></td>
</tr>
</tbody>
</table>

(ii) Include the missing 3-point moving average figures on the graph below. (2 marks)

[Graph showing children's attendance and 3-point moving average over the months from January to October.]
Question 2 (continued)

(b) Determine the equation of the trend line for the 3-point moving average of the number of children attending the child care centre. (Give your numbers to one decimal place.)

\[ y = 3.0x + 46.8 \]  
\[ N = 3.0t + 46.8 \]  
\[ t = \text{months} \]

(2 marks)

(c) Use your equation from part (b) to forecast the number of children that would be attending in October of the (same) year. (Give your answer to the nearest whole number.)

\[ t = 10 \]
\[ N = 3.0 \times 10 + 46.8 \]
\[ = 76.8 \text{ children} \]
\[ \therefore 77 \text{ children} \]

(2 marks)

(d) The child care centre manager finds that they cannot cater for more than 100 children on a regular basis. Showing algebraic workings, predict in which month this could occur for the first time.

\[ N = 100 \]
\[ 100 = 3.0t + 46.8 \]
\[ 53.2 = 3.0t \]
\[ t = 17.73 \text{ months} \]
\[ \therefore 18^{\text{th}} \text{ month: first day of June} \]
\[ \text{the following year.} \]

(3 marks)

(e) Consider your answers to parts (c) and (d). Comment on the reliability of your answers.

C) unreliable because the result is extrapolated, even though it is within the bounds of the original dataset.

D) unreliable, it is well outside the data range and it is unknown what the result could be in 18 months.
Question 3 (approximately 17 minutes)

Sam runs a small seaside shop. He is interested to see if the number of cold drinks he sells is dependent on the temperature of the day.

Sam records the number of cold drinks (C) he sells against the temperature (T) of the day for a period of nine days. The data he collected is shown in the table and the graph below.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Cold drinks sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>22</td>
<td>142</td>
</tr>
<tr>
<td>23</td>
<td>29</td>
</tr>
<tr>
<td>24</td>
<td>37</td>
</tr>
<tr>
<td>25</td>
<td>44</td>
</tr>
<tr>
<td>30</td>
<td>62</td>
</tr>
<tr>
<td>32</td>
<td>68</td>
</tr>
</tbody>
</table>

Sam found the equation of the **line of best fit** and the **correlation coefficient** for this data to be:

\[ C = 3.01 \, T - 21.13 \quad r = 0.3941 \]

(a) Explain the gradient and the y-intercept in context of the variables Sam investigated. Comment on their relevance. (3 marks)

**gradient = 3.01**, this is how much the sales of cans increase per degree of increase in temperature.

**y-intercept = -21.13**, it refers to the number of cans sold when the temperature is 0°C. This is not relevant as our data range and cannot have a negative number of cans sold.

(b) Write a conclusion based on the correlation coefficient. (2 marks)

There is a weak positive correlation between the variables. As the temperature increases there is a little evidence to suggest that the number of cans sold also increases.

**Question 3 continues.**
Question 3 (continued)

(c) The residuals plot for this linear model and the point (22, 96.9) are shown below.

(i) Interpret the **residual point** (22, 96.9), in terms of the variables being investigated.

When the temperature is 22°C, the actual number of cans sold is 96.9, more than predicted by the model.

(ii) Explain the effect that this point value (22, 142) has on the linear model Sam found.

All other values have a negative residual. This indicates the line of best fit does not reflect the trend line of most of the data.

(d) **Remove** the point (22, 142) from the original data point opposite and determine the:

New linear equation: $C = 3.83T - 54.58$

New correlation coefficient: $r = 0.9909$

(e) Prepare a scaled **residual plot** for the new linear model you have found in in part (d).

Question 3 continues.
On the basis of your answers to parts (d) and (e), state whether or not the linear equation you have found in (d) models this data well or otherwise. Give reasons for your choice.

$r = 0.9909$ shows a strong positive correlation indicating there is a lot of evidence to suggest that as temp. rises, number of cans of soft drink sold increases.

The residual plot shows a random distribution of points with values roughly evenly distributed between the positive and negative, indicating a good fit. Also residual values reasonably small.

After removing the outliers the new model fits the data well.

More data or a check on the result for 22°C could also be considered if wish to confirm how good model is.

$r^2 = 0.9817$

About $98.17\%$ of the variation in number of drinks sold can be associated to the variation in temp.
Question 4 (approximately 9 minutes)

The amount of cherries produced in Tasmania, in tonnes, by the end of each year, from 2008 to 2011 are shown in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cherries (tonnes)</td>
<td>3 150</td>
<td>4 300</td>
<td>5 450</td>
<td>6 600</td>
</tr>
</tbody>
</table>

(a) If this pattern was to continue, predict how many cherries were produced in 2013. (1 mark)

\[ a = 3150, \quad 2013 \quad (n = 6) \quad d = 1150 \]

\[ T_n = a + (n-1)d \quad 8900 \quad \text{cherries} \]

(b) Write an arithmetic sequence rule that describes the amount of cherries produced. (1 mark)

\[ T_n = a + (n-1)d \]

\[ = 3150 + (n-1)1150 \]

\[ = 1150n + 2000 \]

(c) Using the arithmetic sequence from part (b), algebraically determine at the end of which year the amount of cherries produced would first exceed 12 000 tonnes. (3 marks)

\[ 12000 = 2000 + 1150n \]

\[ 1150n = 10000 \]

\[ n = 8.69 \quad \text{years} \]

\[ 9^{\text{th}} \text{ year before exceeds } \]

\[ \therefore 9^{\text{th}} \text{ year } \]

(d) Defining the terms you use, write a difference (recursive) equation that could also be used to predict the amount of cherries produced each year. (2 marks)

\[ T_{n+1} = T_n + 1150 \]

\[ T_1 = 3150 \]

(e) Assuming this arithmetic sequence continues, for the years 2008 to 2017, inclusive, determine the total amount of cherries produced over that time period. (2 marks)

\[ S_n = \frac{n}{2} \left( 2a + (n-1)d \right) \]

\[ = \frac{10}{2} \left( 2 \times 3150 + 9 \times 1150 \right) \]

\[ = 83250 \quad \text{cherries} \]

or from calculator in Sum

\[ n = 10 \]

\[ S_{10} = 83250 \quad \text{cherries} \]
Question 5 (approximately 5 minutes)

Ahmed and Jasmin start working at the same time. They both start on salaries of $55 000.

Over a ten year period, Ahmed’s salary increases by $4 000 each year, whilst Jasmin’s salary increases by 6.0% each year.

(a) What are their salaries in the second year?  

\[ \text{Ahmed} = 55000 + 4000 = 59000 = A_2 \]
\[ \text{Jasmin} = 55000 \times (1.06) = 58300 = S_2 \]

(b) Using appropriate sequence formulae and other supporting evidences, compare Ahmed’s and Jasmin’s salaries over a ten year period.  

Note from here there were several different ways to answer this question.

\[ A_n = 55000 + 4000(n-1) \quad \text{Ahmed} \]
\[ S_n = 55000 \times (1.06)^{n-1} \quad \text{Jasmin} \]

\[ n = 8 \quad A_8 = 83000 \quad S_8 = 82700 \quad \text{From calculator} \]
\[ A_9 = 87000 \quad S_9 = 87661.64 \]
\[ A_{10} = 91000 \quad S_{10} = 92921.34 \]

\[ \text{Sum: } \text{Ahmed}_10 = 780000 \quad \text{Jasmin}_10 = 724943.72 \]

Ahmed’s salary is more than Jasmin for years 2 to year 8.

In year 9 and 10, Jasmin earns more per year.

Over a 10 year period, Ahmed has been paid a total of $505.63 more than Jasmin.

Other possibilities were to draw graphs.
Question 6 (approximately 9 minutes)

Sue runs and cycles as part of her weekly exercise program.

The distance Sue runs each week can be modelled by the difference equation:

\[ R_{n+1} = 0.75 \times R_n + 5.0 \quad \text{where } R_1 = 9.0 \text{ km} \]

(a) Determine the distance that Sue will run in week 2. (Give your answer to one decimal place).

\[ R_2 = 0.75 \times 9.0 + 5.0 = 11.75 \text{ km} = 11.8 \text{ km} \]  

The distance Sue cycles each week can be modelled by the difference equation:

\[ C_{n+1} = 0.90 \times C_n + 4.0 \quad \text{where } C_1 = 6.0 \text{ km} \]

(b) **Graph** the distances that Sue runs and cycles for weeks 1 to 7 on the grid below.

(3 marks)

\[ \text{Graph symbols: } \bigcirc = \text{run} \]
\[ \boxtimes = \text{cycle} \]

Question 6 continues.
Question 6 (continued)

(c) Use the graph for weeks 1 to 7 and data from your calculator that extends beyond week 7 to describe the trend in these two forms of exercise that Sue undertakes. What are the maximum distances she runs and cycles, and when approximately will these occur? (3 marks)

Trend for both = increases more at start and then increase gets smaller and smaller.

From calculator: Sue runs a maximum of 26 km and this occurs about week 12 (19.53 km) week 20 = 19.95 km.
Sue cycles a maximum of 40 km and this occurs about week 42 (39.55 km).

Week 4 approx the same distance although cycles slightly less. Week 5 cycles more than ours.

Sue decides to cycle a constant distance of 45 km each week.

A new difference equation (shown below), could be used to model this new scenario:

\[ C_{n+1} = 0.90 \, C_n + k \quad \text{where} \quad C_1 = 45 \, \text{km} \]

(d) Determine the value of 'k' and write the new difference equation. (2 marks)

\[ \begin{align*}
45 &= 0.9 \times 45 + k \\
\implies k &= 45 - 40.5 \\
\implies k &= 4.5 \, \text{km}
\end{align*} \]

\[ C_{n+1} = 0.90C_n + 4.5 \]
At the beginning of the first year of a scientific study, the population of a parrot species was 150. At the beginning of the fifth year of this study, the population of parrots had decreased to 48.

A graph of the population of parrots over this time period is shown below.

(a) Show that a geometric sequence rule that could be used to describe how the population changes is:

\[ t_n = 150 (0.75)^{n-1} \]

\[ T_1 = 150 \quad T_5 = 48 \quad a = 150 \]

\[ T_5 = a \cdot r^4 \]

\[ 48 = \frac{150}{r^4} \]

\[ r^4 = \frac{48}{150} = 0.32 \]

\[ r = \pm 0.75212 \quad \text{the homework graph} \]

\[ r = 0.75 \, \text{to 2 decimal places} \quad \therefore a = 150 (0.75)^{n-1} \]

(b) What is the annual percentage decrease in the population? (1 mark)

\[ 25\% \, \text{decrease} \quad (100\% - 75\%) \]

\[ (1 - 0.75) \]

(c) Use the geometric sequence rule from part (a) to predict at the beginning of which year the population would become extinct, i.e. when there are less than 1.0 birds. (2 marks)

From calculator

\[ n = 18 \quad T_{18} = 1.12 \]

\[ n = 19 \quad T_{19} = 0.8457 \]

\[ \therefore \text{year 19 there is less than 1 bird} \]

\[ \therefore \text{extinct year 19} \]
Question 7 (continued)

When the parrot population had decreased to 48, it was decided to add 30 extra parrots from a captive breeding program, at the beginning of each year. Scientists predicted that this would also result in an annual 18% population decrease.

(d) Identify ‘r’ in the first order difference (recurrence) equation below that will predict the population of the parrots at the beginning of each year. (1 mark)

\[ P_{n+1} = r \cdot P_n + 30, \text{ where } P_1 = 48 \text{ parrots} \]

\[ P_{\text{new}} = 0.82 \cdot P_n + 30 \]

\[ r = 1 - 0.18 \]

\[ = 0.82 \]

(e) Use this equation to describe the change in population in parrots over the next 25 years. Approximately at what number will the population of parrots stabilise? (3 marks)

Over the next 25 years, the population will increase rapidly at first, then each year the increase is slightly smaller until increase less than 0.5. At approximately 166 parrots the population stabilises.

When the scientists started adding 30 birds each year, they found that the population decrease was more than 18%, with the population stabilising at 140 parrots.

(f) Based on this information, determine the actual ‘r’ figure and the difference equation. Determine the actual annual percentage population decrease that occurred. (4 marks)

\[ 140 = r \cdot (140) + 30 \quad P_n = 140, P = 140 \]

\[ 140 \cdot r = 110 \]

\[ r = \frac{110}{140} \]

\[ = 0.7857 \]

Difference equation: \[ P_{\text{new}} = 0.7857 \cdot P_n + 30 \quad P_1 = 140 \]

Actual annual percentage population decrease:

\[ 1 - 0.7857 \]

\[ = 0.2143 \]

\[ = 21.43\% \text{ per year.} \]
Question 8 (approximately 8 minutes)

Ivana wants to invest $20,000 for three years. Two banks offered her the following investment options:

**Bank A** 2.52% p.a. (nominal) compounded monthly

**Bank B** 2.55% p.a. (nominal) compounded half yearly

(a) Using effective interest rates to compare these rates, determine with which bank she should invest.

- **Bank A**
  
  \[ \text{Effective Interest Rate} = \frac{1 + \frac{i}{n} - 1}{\frac{i}{n}} \]

- **Bank B**
  
  \[ \text{Effective Interest Rate} = \frac{1 + \frac{i}{2} - 1}{\frac{i}{2}} \]

\[ n = 12 \quad \text{and} \quad n = 2 \]

\[ \text{APR} = 2.52\% \quad \text{and} \quad \text{APR} = 2.55\% \]

\[ \text{Eff.} = 2.524981\% \quad \text{and} \quad \text{Eff.} = 2.56675\% \]

\[ \text{or use} \quad E = (1 + i)^n - 1 \]

- **Bank B has a higher effective interest rate.**

(b) If Ivana bought shares with the $20,000 and sold them, 3 years later for $21,952 determine how much extra money she would have made as opposed to investing her money with **Bank B** for 3 years.

\[ A = P (1 + i)^n \]

\[ A = 20,000 \cdot (1 + 0.0255)^6 \]

\[ n = 3 \times 2 \]

\[ P = 20,000 \]

\[ \text{Extra money} = 21,952 - 21,579.60 \]

\[ = 372.40 \]

(c) If instead, Ivana had invested her $20,000 at 4.40% p.a. compounded annually, generate a recurrence (difference) equation representing this situation.

\[ T_{n+1} = (1.044)T_n \quad T_0 = 20,000 \]
**Question 9** (approximately 7 minutes)

‘Payday’ loans are **simple interest** loans that are taken out over short periods of time. The simple interest rates on ‘Payday’ loans are **very high**.

Bill took out a ‘Payday’ loan for $1 000 on 1 March. He repaid the loan with $1 220 on 4 May.

(a) Showing **algebraic workings**, determine the simple interest rate (p.a.) for this loan. (4 marks)

\[
\begin{align*}
I &= PRT \\
220 &= 1000 \times R \times \frac{64}{365} \\
R &= \frac{220 \times 365}{1000 \times 64} \\
&= \frac{1.2546}{64} \\
&= 0.0197 \text{ or } 1.97\% \text{ p.a.} \\
\text{(correct to 2 decimal places)}
\end{align*}
\]

(b) If, instead, Bill had borrowed $1 000 at a lower simple interest rate of 5.9% p.a. between 1 March and 4 May, determine how much interest he would have saved. (3 marks)

\[
\begin{align*}
I &= PRT \\
&= 1000 \times 0.059 \times \frac{64}{365} \\
&= \$10.32 \\
\text{Saved} &= 220 - 10.35 \\
&= \$209.65
\end{align*}
\]
Question 10 (approximately 10 minutes)

Sophie wants to save up to buy a new car for $25 000 in five years time.

To do this Sophie makes regular deposits of $290 per month into an account that pays 3.20% p.a. interest compounded monthly.

(a) How much will Sophie have saved after five years? (3 marks)

\[ n = 5 \times 12 = 60 \]
\[ \text{or } F = 290 \left( 1 + \frac{0.032}{12} \right)^{60} \left( \frac{0.032}{12} \right) \]
\[ i = 3.2\% \]
\[ PV = 0 \]
\[ P \times t = -290 \]
\[ FV = 18892.37 \]
\[ P \times y = 12 \]
\[ C \times y = 12 \]

(b) Sophie currently owns a car valued at $9 000. If this car depreciates at 15% p.a. on a reducing balance, determine its value after five years. (3 marks)

\[ A = P \times (1 - i)^n \]
\[ = 9000 \times (1 - 0.15)^{15} \]
\[ = 3993.35 \]
(c) After five years Sophie is able to sell her car for $3,500. Even with this money, she needed to have increased the amount she deposited every month from the $290 in part (a) in order to have $25,000 after five years.

Determine the extra amount that Sophie would need to deposit every month in order to have a total of $25,000 in five years. (4 marks)

\[
\text{Amount required} = \$25,000 - \$3,500 \\
= \$21,500
\]

Begin

\[ \begin{align*}
N &= 60 \\
I% &= 3.2 \\
PMT &= -330.03 \\
PV &= 0 \\
PMT &= ?? \\
FV &= 21,500 \\
P/y &= 12 \\
c/y &= 12 \\
\end{align*} \]

\[
\therefore \text{require } \$330.03 \text{ each month} \\
\therefore \text{extra amount} = \$330.03 - \$290 \\
= \$40.03
\]
Question 11 (approximately 11 minutes)

A person borrows $320\,000 in order to buy a house. They repay this at an interest rate of 6.80\% \text{ p.a.} compounded monthly over 25 years.

(a) Show that the monthly repayment figure over this 25 year period is $2\,221 to the nearest dollar. \hspace{1in} (2 \text{ marks})

\[ N = 25 \times 12 = 300 \]
\[ I\% = 6.8 \]
\[ PV = 320\,000 \]
\[ P = R \left(1 - \left(1+\frac{I}{12}\right)^{-300}\right) \]
\[ 320\,000 = R \left(1 - \left(1+\frac{0.068}{12}\right)^{-300}\right) \]
\[ \frac{320\,000}{\left(1+\frac{0.068}{12}\right)^{300}} \]
\[ \text{PMT} = 2221.03 \]
\[ FV = 0 \]
\[ P/Y = 12, \ C/Y = 12 \]
\[ \therefore \ $2221 \ to \ closest \ dollar. \]

(b) What are the total repayments over the first five years? \hspace{1in} (2 \text{ marks})

\[ \text{Total Paid} = 5 \times 12 \times 2221.03 \]
\[ = 133,261.80 \quad \text{(currency units)} \]

After five years the interest rate decreases to 5.20\% \text{ p.a.} compounded monthly.

(c) They have decided to keep paying $2\,221 a month after the introduction of this new interest rate. How long from this point will it take to fully repay the loan? \hspace{1in} (3 \text{ marks})

\[ N = \frac{300 - 5 \times 12}{240} \]
\[ I\% = 6.8 \]
\[ PV = 320\,000 \]
\[ PMT = -2221.03 \]
\[ FV = 0 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PV = $290\,962.36 \]
\[ \text{after 5 years.} \]

\[ N = 193.940 \text{ months} \]

Question 11 continues.
(d) Compared to the original loan conditions, how much money is saved by using the lower interest rate after five years in part (c).

\[
\text{Total paid} = 300 \times 2221.03 \\
= \$666309
\]

\[
\text{Total paid} = 60 \times 2221.03 + 193.94 \times 2221.03 \\
= \$564008.35
\]

\[
\text{Saved} = \$666309 - \$564008.35 \\
= \$102300.64
\]
Part 4 - Trigonometry

Question 12 (approximately 6 minutes)

The dimensions of a block of land are shown below.

![Diagram of a block of land with dimensions labeled: AB = 1400 m, AC = 1300 m, BC = 1900 m, and angle at A = 50°.]

(a) Show that the length of AB is 1.145 m. (2 marks)

\[ d^2 = a^2 + b^2 - 2ab \cos \theta \]
\[ = 1400^2 + 1300^2 - 2 \times 1400 \times 1300 \times \cos (50°) \]
\[ = 13,102,533.6 \]
\[ \therefore d = \sqrt{13,102,533.6} = 1144.66 \]

(b) Determine the area of the block of land. (4 marks)

\[ \text{Area}_1 = \frac{1}{2} \cdot a \cdot b \cdot \sin \theta \]
\[ = \frac{1}{2} \times 1400 \times 1300 \times \sin 50° \]
\[ = 69,710.08 \text{ m}^2 \]

\[ \text{Area}_2 = \sqrt{s(s-a)(s-b)(s-c)} \quad s = \frac{a+b+c}{2} \]
\[ = \sqrt{2122.5 \times (2122.5-1400) \times (2122.5-1300) \times (2122.5-1145)} \]
\[ = 2122.5 \times 0.740 \]
\[ = 652.57499 \text{ m}^2 \]

Total area = 13,496.754 m²
\[ \approx 13,497.00 \text{ m}^2 \]
Two bird watchers observe an eagle that is flying due east of them at the same time.

Person A observes the eagle at an angle of elevation of 27°. Person B, who is 200 m due east of person A, observes the eagle an angle of elevation of 48°.

(a) Complete the diagram illustrating this in the space below. (2 marks)

(b) Use trigonometry to determine the height of the eagle above the ground. (4 marks)

\[
\frac{x}{\sin 27°} = \frac{200}{\sin 48°} \\
\frac{x}{\sin 27°} = \frac{200 \cdot \sin 27°}{\sin 48°} = 188.39 \text{ m}
\]

\[x = 188.3 \text{ m above the ground}\]
Question 14 (approximately 9 minutes)

Two fishing boats leave port at the same time. The Cindy Lou travels on a bearing of N 50° E for 26 km. The Stig Larson travels 23 km on a different bearing, as shown in the diagram below. After two hours, the boats stop and are 33 km apart.

(a) Use the above information to complete the diagram below illustrating this situation after two hours. (2 marks)

(b) Use trigonometry to show that the angle (θ) formed between the two boats is 84.4°. (3 marks)

\[
\cos \theta = \frac{26^2 + 23^2 - 33^2}{2 \times 26 \times 23}
\]

\[
= \frac{676 + 529 - 1089}{588}
\]

\[
= \frac{124}{588}
\]

\[
= 0.2104
\]

\[
\therefore \theta = 84.4^\circ
\]

\[
= 84.4^\circ
\]
The Cindy Lou then travels another 12 km on the same bearing (N 50° E) before stopping, as shown below. (The Stig Larson remains where it was.)

Determine the distance between the two boats and the bearing that the Cindy Lou is now from the Stig Larson. (4 marks)

\[
d^2 = 38^2 + 23^2 - 2 \times 38 \times 23 \times \cos(84.4°)
\]
\[
= 1802.43
\]
\[
\therefore d = \sqrt{1802.43}
\]
\[
= 42.45 \text{ km}
\]

\[
\cos \theta = \frac{23^2 + 42.45^2 - 38^2}{2 \times 23 \times 42.45}
\]
\[
= 0.4544
\]
\[
\therefore \theta = 62.97°
\]

\[
d = 62.97° - 45.6°
\]
\[
= 17.37°
\]
\[
\therefore \text{Bearing } N 17.37° E
\]
Question 15 (approximately 15 minutes)

A cargo ship sailed from Fremantle, Western Australia (32°S, 116°E) at 10:00 pm on 14 October, taking the **shortest possible route** to Colombo, Sri Lanka (7°N, 80°E).

(a) What is the standard time difference between Fremantle and Colombo? (2 marks)

<table>
<thead>
<tr>
<th>Location</th>
<th>Time Difference</th>
<th>UTC Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fremantle</td>
<td>+8 UTC</td>
<td>(116/15 = 7.73h)</td>
</tr>
<tr>
<td>Colombo</td>
<td>+5 UTC</td>
<td>(80/15 = 5.33h)</td>
</tr>
</tbody>
</table>

Fremantle is 3 hours ahead of Colombo.

(b) In Fremantle, a person makes a phone call to their friend in Colombo. If they want to contact their friend in Colombo, between 8:00 am and 6:00 pm, Sri Lankan time, between what Western Australian times should they make this phone call? (2 marks)

<table>
<thead>
<tr>
<th>Time</th>
<th>Conversion to Fremantle</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 am</td>
<td>5:00 am</td>
</tr>
<tr>
<td>9:00 am</td>
<td>4:00 am</td>
</tr>
<tr>
<td>11:00 am</td>
<td>1:00 pm</td>
</tr>
<tr>
<td>6:00 pm</td>
<td>3:00 pm</td>
</tr>
</tbody>
</table>

On the same day, until 9:00 pm.

(c) Determine the shortest **distance** between Fremantle and Colombo. Give your answer to the nearest nautical mile. (4 marks)

\[
D = 600 \theta \text{ nm}
\]

\[
\cos \theta = \sin(-32) \sin(7) + \cos(-32) \cos(7) \cos(36)
\]

\[
= 0.9639
\]

\[
\theta = 51.946^\circ
\]

\[
D = 60 \theta = 60 \times 51.946 = 3116.81 \text{ NM}
\]

Question 15 continues.
Question 15 (continued)

(d) If the cargo ship sailed at an average speed of 21 knots determine its arrival date and time in Colombo, Sri Lanka. (4 marks)

\[
\text{Speed} = \frac{\text{dist}}{\text{time}}
\]

\[
\text{Time} = \frac{317}{21} = 14.847 \text{ hours}
\]

\[
= 14 \text{ hours } 53 \text{ mins}
\]

\[
= 6 \text{ days } 4 \text{ hours } 25 \text{ mins}
\]

Zone time difference = -3 hours.

\[
\text{ETA} = 10 \text{pm Oct } 14^{th} + 6 \text{days } 4 \text{ hours } 25 \text{ mins} - 3 \text{ hours}
\]

\[
= 10 \text{pm Oct } 20^{th} + 1 \text{ hour } 25 \text{ mins}
\]

\[
= 11:25 \text{pm Oct } 20^{th}
\]

Colombo, Sri Lanka (7°N, 80°E) is on the same line of latitude as Georgetown, Guyana.

(e) Georgetown, Guyana is 15 230 km due west of Colombo. Determine the longitude and hence the position coordinates of Georgetown. Give the position coordinates to the nearest degree. (3 marks)

\[
D = \frac{2\pi R \cos \theta \times \phi}{360}
\]

\[
D = 7^\circ \ (7^\circ N)
\]

\[
15230 = \frac{2\pi \times 6371 \times \cos 7^\circ \times \phi}{360}
\]

\[
\theta = ?
\]

\[
R = 6371 \text{ km}
\]

\[
15230 = 110 \cdot 366 \theta
\]

\[
\theta = \frac{15230}{110 \cdot 366} = 137.995^\circ = 138^\circ
\]

\[
\therefore \text{Longitude difference } = 138^\circ
\]

\[
\therefore \text{Longitude } = 138 - 80
\]

\[
= 58^\circ \therefore 58^\circ W
\]

\[
\therefore \text{Georgetown } (7^\circ N, 58^\circ W)
\]

\[\text{Diagram showing west orientation} \]

MTG315115
The network diagram below shows the flow capacity, in kilolitres (kL) per minute, in water pipes connecting points A to G.

(a) Identify the source and the sink of this network.

Source: ........................................ (1 mark)
Sink: ...........................................

(b) Without calculating the maximum flow, briefly explain why some of the 48 kL per minute of water that could flow from the source is not able to be carried by the irrigation system to the sink.

Because the flow into the sink has a smaller capacity. Not all pipes have the same capacity hence may restrict the flow.

(c) Use the ‘minimum cut = maximum flow’ method to determine the maximum flow in this network. Show your workings on the network diagram above.

Maximum flow: .......... 44 kL per minute

(d) If only one of the pipes could be modified to improve the water flow, which pipe could this be, and by how much could this pipe be increased? Give two possible solutions.

Pipe F to G increased by 2 kL per minute
(new capacity 46 kL/min)

Pipe E to G increased by 2 kL/min
(new capacity 46 kL/min)

Pipe B to G increased by 2 kL/min
(new capacity 4.6 kL/min)
**Question 17 (approximately 11 minutes)**

A market gardener has an irrigation system consisting of a series of pipes with a water inlet at vertex A and outlets at vertices B to I.

(a) Starting at vertex A, and then walking to vertex B and then onto vertex C a person walks along all of the pipe sections once and once only. (2 marks)

(i) What is the name given to such a path?

**Eulerian Path**

(ii) Detail a possible path that the person could take. (many different answers must start at A)

\[ A \rightarrow B \rightarrow C \rightarrow I \rightarrow A \rightarrow H \rightarrow G \rightarrow F \rightarrow H \rightarrow I \rightarrow F \rightarrow E \rightarrow D \rightarrow C \]

(b) A student notes that the addition of one more pipe would make it possible to enable a circuit walk starting and finishing at vertex A. (3 marks)

(i) On the network diagram below, draw in an additional pipe (edge) that would enable this to occur.

(ii) Starting and finishing at vertex A, detail such a circuit.

\[ A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow H \rightarrow F \rightarrow I \rightarrow C \rightarrow A \rightarrow I \rightarrow H \rightarrow A \]

(many different answers)

*Question 17 continues.*
Question 17 (continued)

The market gardener expands their watering system by installing additional water outlets at J, K, L and M.

(c) Using the diagram below, where the lengths of edges are in metres, determine the **minimum distance** between A and L. Describe this shortest path. 

![Diagram with distances]

Shortest path taken from A to L: \(A - H - F - E - J - L\)

Distance of path: \(369 \text{ m} \) \((110 + 105 + 42 + 65 + 47)\)

(d) The market gardener decides to replace all these pipes with new pipe system. On the diagram below draw the **minimum spanning tree** required to do this and determine the **total length of pipe** needed.

![Diagram with distances]

Total length of pipe needed: \(98 + 100 + 45 + 55 + 75 + 65 + 42 + 49 + 59 + 62 + 47 + 85\)

\[= 783 \text{ m}\]
Question 18 (approximately 11 minutes)

The activities table and the partially completed project network graph below show activities ‘A’ to ‘J’, their predecessor(s) and the time that each activity takes, in days.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time (days)</th>
<th>Predecessor(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>A, B</td>
</tr>
<tr>
<td>E</td>
<td>11</td>
<td>B</td>
</tr>
<tr>
<td>F</td>
<td>7</td>
<td>C</td>
</tr>
<tr>
<td>G</td>
<td>12</td>
<td>D, E</td>
</tr>
<tr>
<td>H</td>
<td>10</td>
<td>D, E</td>
</tr>
<tr>
<td>I</td>
<td>3</td>
<td>F</td>
</tr>
<tr>
<td>J</td>
<td>7</td>
<td>G</td>
</tr>
</tbody>
</table>

(a) Use the activities table to complete the project network graph below, drawing and labelling all of the edges and vertices. (3 marks)

Question 18 continues.
(b) A dummy activity, \((X, 0)\), starts at the end of activity B. Explain why this dummy activity is used on the this project network. (2 marks)

As activity D requires both activities A and B to be completed, and activity E only requires activity B to be completed.

(c) Identify the critical path of the project network and hence the minimum time to complete the project. To obtain full marks, numbers must be added to the project network opposite. (3 marks)

Critical path: Start - B - E - G - J - Finish

Minimum time: 42 days

(d) The duration of activity F is delayed by 'x' days. For what value of 'x' does the critical path determined in part (c) remain critical and the minimum time remain the same? (3 marks)

Float time = LFT - activity time - EST
activity F = 39 - 7 - 9 = 23 days.
up to and including 23 days activity F can be delayed by for the critical path to remain unchanged, and the minimum completion time of 42 days to remain the same.
**Question 19** (approximately 6 minutes)

Four people, Alain, Beth, Christine and Dion are asked to give a quote (in $ per hour) to do four jobs, J1, J2, J3 and J4. The hourly rates that they quote are shown in table below.

<table>
<thead>
<tr>
<th></th>
<th>J1 ($)</th>
<th>J2 ($)</th>
<th>J3 ($)</th>
<th>J4 ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alain</td>
<td>44</td>
<td>49</td>
<td>48</td>
<td>53</td>
</tr>
<tr>
<td>Beth</td>
<td>45</td>
<td>51</td>
<td>50</td>
<td>46</td>
</tr>
<tr>
<td>Christine</td>
<td>43</td>
<td>52</td>
<td>47</td>
<td>52</td>
</tr>
<tr>
<td>Dion</td>
<td>46</td>
<td>48</td>
<td>46</td>
<td>55</td>
</tr>
</tbody>
</table>

(a) Use the **Hungarian algorithm** method in order to make an assignment that will result in these jobs being completed using the cheapest possible hourly rates. (4 marks)

(b) Use your answer from part (a) to allocate Alain, Beth, Christine and Dion to the jobs. (1 mark)

Alain = J2  Christine = J1
Beth = J4  Dion = J3

(c) If the four people all worked for 8 hours, what would be the **minimum total cost**? (1 mark)

\[ 43 \times 8 + 49 \times 8 + 46 \times 8 + 46 \times 8 = 8 \times 184 \]

\[ = 8 \times 184 = \$1472 \]