PART 1 – Criterion 4

Examiners found that the main areas that candidates and teachers need to address are:

• Poor understanding and interpretation of the formulae sheet particularly with regards to residuals.
• The reasonableness of their answers, making sure units are included and values not rounded until the final answer is given.
• Ensure that the pro-numerals provided are used and not the generic variables of y and x.
• Answers to the worded questions also need to include justifying choices and selections.
• Written material should focus on answering the question and not repeating elements of the question or transcribing from the formulae sheet.

Teachers need to include more exploration of residuals and what constitutes a good model.

Question 1

a) This question was very poorly understood by most candidates.

b) This question was reasonably well done by most candidates. Some candidates neglected to use all of the information given to justify a conclusion or conclusions were simply omitted. There was some confusion with the use of words or phrases such as random and no pattern. Many candidates quoted the statements given on the information sheet rather than applying these statements to the question and taking into account context.

Question 2

a) Both parts to this question were done very well by most candidates. Some candidates wrote the answer as an expression instead of an equation.

b) i) This question was poorly done by most candidates. Very few candidates included axes scaled and labelled correctly, three models drawn using correct domains, models labelled (identified), endpoints labelled (or at least shown in a table) and breakeven point indicated.

ii) This question was well done by most candidates. They showed a good level of understanding of the algebraic process required to find a break-even point. Candidates did have difficulty finding the required break-even point if they did not complete the graph in part b) (i). There was some confusion around the number that matched the minimum required for a profit – incorrect rounding.
Question 3

a) This question was reasonably well done by most candidates. Common errors included incorrect rounding and neglecting to use appropriate pro-numerals. Some candidates only wrote the values for \( r \) and \( r^2 \) instead of the equations of models or wrote the values of ‘a’ and ‘b’ from the calculator and not equations.

b) This question was quite poorly done by most candidates. Candidates frequently forgot to consider residuals when deciding on the model. They also inappropriately used \( r \) to justify the validity of a model.

c) This question was reasonably well done by most candidates. Candidates needed to make a statement regarding what the \( r \) value meant in terms of the variables used in order to be award a B.

d) This question was very well done by most candidates.

e) This question was very well done by most candidates.

f) This question was very well done by most candidates at superficial level. They justified the reliability of predictions based on interpolation and extrapolation, but neglected to refer to and use the \( r^2 \) value in their justifications. Often wrote learnt responses were given.

PART 2 – Criterion 5

Candidates appeared to be well prepared for this examination section and knew how to approach the questions asked however there still was a large number of candidate who were poorly prepared and did not know how to approach the problem asked.

Examiners found that the main areas that candidates and teachers need to address are in the area of graphing skills, and reading and interpretation of the questions asked. General communication needs to be improved, with the expected inclusion of units with all answers. Giving the examiner clear guidance in what you are doing is very beneficial. Graphs need to contain the requirements stated in the question (e.g. y-intercept, end point, zero etc.) and must also have labelled axis with units, some scale on each axis and points given as coordinates. The concept of a restricted domain was often ignored and a general graph was given.

When a question starts with the statement **Use calculus techniques**, this infers that the **solution** is required to have an initial statement about max/min occurs when \( \frac{dy}{dx} = 0 \). Clearly distinguishing between the \( y \) function and the derivative function \( \frac{dy}{dx} \)

Make sure you **READ THE QUESTION** so you answer what is being asked not what you think is being asked.

Candidates are encouraged to improve their use of the calculator for graphing.
Question 4

a) This question was quite well done by most candidates. Weaker candidates wrote an expression rather than an equation in part i). Many candidates did not give the units for speed and there was some confusion in part ii) in what was required.

b) This question was generally well done by most candidates although some candidates substituted into original equation. There were some calculator errors.

c) This question was quite poorly done by most candidates. Common errors were candidates drew in the line by hand and did not use a ruler to draw the straight line; the scale was missing or inappropriate scale was used; the end points were not labelled; some candidate drew the original equation instead of the derivative equation; plotting several points when only end points needed to draw the line.

Question 5

a) This question was very poorly done by most candidates. Very few candidates could generate the expression for the height (h) of the box and consequently could not find the volume. Some candidates found the wrong h and ‘fudged’ to get the result rather than following through. Many candidates made little or no attempt at this question.

b) This question was quite well done by most candidates. Common errors were that candidates forgot to make a statement about max occurs when \( \frac{dV}{dx} = 0 \). Clear and logical setting out was often missing in candidates solutions. Some candidates were not clearly distinguishing between the function (V) and the derivative function (\( \frac{dV}{dx} \)). Both solutions were not given with some candidates dividing through by x which meant they lost the second solution or not recognising that \( x = 0 \) was a possible solution. Many candidates did not clearly distinguish between the minimum and maximum x solutions. Most candidates found the maximum volume correctly but units were often not included.

c) This question was very poorly done by most candidates. Many candidates had difficulty coping with the function being in fractional form, \( \frac{dH}{dx} \). Some candidates got a little confused with the two forms of the equation being given. Common errors were that candidates forgot to make a statement about max occurs when \( \frac{dH}{dx} = 0 \). Clear and logical setting out was often missing in candidates solutions. Some candidates were not clearly distinguishing between the function (H) and the derivative function (\( \frac{dH}{dx} \)). Some candidates solved problem by losing the denominator of the fractions. Both solutions were not given and many candidates did not clearly distinguish between the minimum and maximum x solutions. Some candidates did not show the substitution and simply used what was given in the question. Many candidates were unable to find the derivative and used their calculator to find \( \frac{dH}{dx} \).

c) This question was quite poorly done by most candidates. A number of candidates did not attempt this question which was surprising given they could have used their calculators. Many candidates did not clearly indicate the shape of the graph; it looked more like a quadratic rather than a cubic.
The scale was missing or inappropriate scale was used; the end points, intercepts and turning points were not labelled; Some candidates did not limit between 0 and 240.

Question 7

a) This question was quiet well done by most candidates. The most common problem was that candidates did not find the correct expression for the x column
b) This question was quite well done by most candidates. The main error occurred due to incorrect expression in the x column from the table which meant b)ii) was not done correctly. bii) was very well done but a number of candidates went on to state the maximum power which was not asked for in this question.

PART 3 – Criterion 6

Examiners agreed that the section was well within the difficulty and scope of the course, but results in this section were quite polarised. The main areas candidates need to focus on is:

- Communication, rounding, the general reading of the question and construction of diagrams. Candidates are advised to take the time to understand what the question is asking before commencing.
- Candidates often had difficulty converting angles from decimals to degree and minutes, and converting time from hours and minutes to decimals to use in calculations. Many calculator entry errors were found.
- Correctly choosing the appropriate formula to solve the problem regularly swapping small and greater circle formulas, some candidates showed a lack of understanding of these formula. There was some concern that many candidates did not known a knot was nautical mile per hour. The angles on the diagram need to be clearly shown.

Question 8

a) This question was quiet well done by most candidates. They often could find time zones correctly but could not apply these to the question. Some candidates neglected to round time zones to the nearest hour.
b) This question was reasonably well done by most candidates, with a high number of C grades awarded. There were some errors in Θ calculations and many could not apply the formula. There were many errors in travel time calculations, with 7 hours and 5 minutes often written incorrectly as 7.05 hours.

Question 9

a) This question was well done by most candidates. Some candidates forgot to subtract the radius of the earth to find the altitude, or wrote 270km as 0.270km, or attributed the 270km to height rather than the distance to the horizon.
b) This question was very well done by most candidates.
c) This question was quite well done by most candidates. The main error was in using inappropriate formula – tried to calculate in kilometres rather than in nautical miles.

Question 10

a) This question was quite poorly done by most candidates considering that it was a very common type of question given in past exams. Many candidates used the wrong formula.
b) This question was quite poorly done by most candidates. Many candidates swapped the longitude and latitude values in the formula. Other common errors include omission of Cos in the formula, angle difference calculations and using the wrong formula.
c) This question was very well done by most candidates.

Question 11

a) This question was well done by most candidates. Some candidates could not correctly transpose the formula.
b) This question was quite poorly done by most candidates. A common error was that the bearing was quoted from A rather than from B. Many candidates were able to calculate an appropriate angle but were not able to write this as a bearing.

c) This question was very well done by most candidates.

Question 12

This question was quite well done by most candidates considering it was an A standard question. Some candidates incorrectly assumed the structure contained a right-angle and there were few calculator errors.

PART 4 – Criterion 7

Examiners found that the main areas that candidates and teachers need to address are the reading and interpretation of the questions asked. General communication needs to be improved, with the expected inclusion of units with all answers. They should also consider the reasonableness of their answers and make appropriate corrections. Candidate’s answers to the worded questions also need to include justifying choices and selections. Also candidates should be encouraged to make concise worded answers. Written material should focus on answering the question and not repeating elements of the question. Candidates are reminded that rulers are a necessary part of their stationary equipment in this subject. There was also some evidence that candidates need to become more proficient with the use of their calculators in this section. Candidates must also provide some Mathematical ‘substance’ in B’ and ‘A’ questions when comments are asked for. Across the questions candidates were inconsistent in their use of significant figures or failed to round correctly.
Question 13

a) This question was quite well done by most candidates. Some candidates demonstrated a poor ability to read scale many read intervals in ‘2’s rather than the required ‘4’ many gave the percentage rather than the number of cars.
b) This question was reasonably well done by most candidates although many candidates gave their answer in percentages, or gave the percentage of cars under the speed limit rather than over the speed limit, or wrote 86% were above the speed limit.
c) This question was quite poorly done by most candidates. Candidates struggled to understand what the interquartile range meant, many wrote Q1-Q3 and didn’t subtract. Very few used km/h. Some candidates thought that the interquartile range was the majority of the data.

Question 14

a) This question was quite well done by most candidates. Candidates misinterpreted what a five figure summary was. Of those who attempted the question a surprising number did it by hand rather than using the calculator.
b) This question was generally well done by most candidates although some candidates drew box and whisker plots on top of the scales. Those who made errors failed to rule their box and whisker plots, not many labelled the scale and some did not label which was Site A and Site B.
c) This question was quite poorly done by most candidates. Candidates should be encouraged to make concise worded answers. Written material should focus on answering the question and not repeating elements of the question. Candidates must also provide some Mathematical ‘substance’ in their answers.
  i) Not many candidates used figures to back up their statements or did not describe how the two sets of data differed. Some candidates referred to the mean rather than using the data from their 5 figure summary, some candidates interchanged the words mean and median struggling to understand the difference.
  ii) Candidates did not conclude which site was best; many rambled without offering any justification. Some assumed that lower wind speeds were better or said one site was more consistent without justifying it with figures. A high number of candidates contradicted themselves between i and ii. Best candidates recognised that the median of site A was equal to the Q3 of site B. Answers needed to be structured as some became incoherent.

Question 15

a) This question was well done by those candidates who attempted the question. However a reasonable number of candidates did not know how to do normal distribution problems and did not attempt this problem. The diagrams that they chose to add were very well done as were the calculator inputs which candidates chose to add on the side. Upper limit needed to be a very large number, not just 100.
b) This question was quite poorly done by most candidates. Of those candidates who attempted this part they really struggled to find a percentage, but did continue to attempt to answer this question. Some left 38.72 although the question asked for a whole number.
c) This question was quite poorly done by most candidates. The examiners were surprised by the number who did not attempt this question or selected more than one even though it was multiple choice.

**Question 16**

a) This question was generally well done by most candidates although the hypothesis statements were quite good some candidates used m instead of \( \mu \); forgot to put in the conditions of their hypothesis or put =0 or confused a greater than and less than sign.

b) This question was quiet well done by most candidates. Many candidates put standard deviation as the probability or did not find the correct probability. Some candidates gave a poorly rounded answer.

c) This question was generally well done by most candidates although some candidates wrote that 2.17% was greater than 5% or wrote 0.02% or although 2% was less than 5% kept the null hypothesis. However, most concluded correctly from their result.

**PART 5 – Criterion 8**

All candidates need to read the questions with care and answer the questions asked. They should also consider the reasonableness of their answers and make appropriate corrections. Rounding appropriately was a major issue. Candidates should be careful of carrying over rounding errors and in Finance all significant figures should be used until the final answer. For questions requiring written answers, candidates should include mathematical reasoning and evidence. Candidates must also recognise where they need to use the TVM/ finance solver on their calculators. Ensuring they provide enough evidence to support their answer, especially when writing information from the calculator. Candidates who didn’t use their calculator were disadvantaged in some questions. Errors also occurred when putting information into the calculator. In particular there was some confusion with using the TVM mode of the calculator of when to use begin and end. However when asked to use an appropriate formula candidates were assigned a t rating if they did not clearly demonstrate the formula used but obtained the answer directly from the TVM/ finance solver part of the calculator. Candidates are encouraged to check their answer from the calculator if they have time.

Communication is important to ensure markers can assess candidate thinking, many candidates need to show more communication. Candidates seem to concentrate their learning on more complex ‘annuity questions’ at the expense of understanding the ‘simpler’ interest and depreciation formula in 17a and 19a and 19b. Greater depth in candidate understanding of depreciation implications for tax would be beneficial. More practice using TVM mode would be advantageous, as some candidates appeared to lack the ability to use TVM mode effectively.

**Question 17**

a) i) This question was quite well done by most candidates. Common errors in i) were 360 days in the year was often used by candidates, although asked to use formula, many candidates did not.

ii) This question was quite poorly done by most candidates. Candidates had difficulty in putting the information into the formula correctly. Calculation of i and n were inaccurate. Candidates
were confused by the compounding over a period of days. Most candidates correctly subtracted the principal to find the interest.

b) This question was quite well done by most candidates. Calculation using TVM was well done, some candidates who tried to use the formula had difficulties identifying the correct i and n and algebraic errors were made. In this question it was advisable to use the calculator.

Question 18

a) This question was quite well done by most candidates. Some candidates tried to use the compound interest formula or the annuities in arrears formula. Some candidates showed correct algebraic substitutions, however then made errors using the calculator.

b) These questions very poorly done by most candidates.
   (i) Many candidates did not seem aware of the correct procedure involved to solve the question. Some candidates were able to determine the present value after the six years, including the one off investment, however could not then determine the end value after 20 years.
   (ii) Many candidates forgot to subtract the one off payment. Some candidates had difficulty finding the interest and this caused confusion and as a result used the Simple Interest formula.

Question 19

a) This question was very well done by most candidates.

b) This question was very poorly done by most candidates. Many candidates were unable to determine the correct depreciation value.

c) This question was quite poorly done by most candidates. The complexities of depreciation for taxation purposes escapes most candidates. A significant number of candidates focussed on the resale value and so made incorrect conclusions. Some candidates did not read the question correctly, referring to depreciation in the fourth year or at the fifth year, rather than ‘over the five years.’ Many candidates did not show any mathematical justification for the intersection point of the two graphs or depreciation values.

Question 20

a) This question was very poorly done by most candidates. A significant number of candidates attempted to use the formula, which cannot work in this situation. General confusion between when to use monthly or fortnightly in the question. Calculator errors included: begin and end, incorrect number of fortights in a year, incorrect n value which should be number of instalment periods, swapped PV and FV.

b) This question was quite poorly done by most candidates. Candidates struggled to figure out the correct approach for the question and having to use TVM mode on the calculator. General confusion between when to use monthly or fortnightly in the question. Calculator errors included: begin and end, use of negative values in the TVM mode, incorrect number of fortights in a year, incorrect n value which should be number of instalment periods, swapped PV and FV.
**Question 1** (Approximately 8 minutes)

The results of a survey of 220 people’s wrist and ankle measurements were recorded to see if there was a correlation between a person’s wrist size and ankle size.

The residual plot below results from a power model being applied to this data.

The coefficient of determination for this model is 0.4500.

![Residual plot](image)

(a) The residual point A (26.6, 1.51) corresponds to the original data point (26.6, 21.4). Use this information and the model described above to estimate the size of a person’s ankle based on a wrist size of 26.6 cm.

\[
\text{Model estimate} = 21.4 - 1.51
\]

\[
= 19.89 \text{ cm}
\]

(b) Justify whether this model is appropriate to use in estimating ankle size from the wrist size.

The residual plot shows random, evenly distributed points above and below the axis. The coefficient of determination is suggesting only 45% variation in ankle size is associated with variation in wrist size. It is an appropriate model to use, but with limitations in its predictive power.
Question 2 (Approximately 16 minutes)

A company manufactures only rabbit hutches and makes a maximum of 40 hutches per month.

Each rabbit hutch costs $60 to make. The fixed cost for running the company is $5 500 per month. Use $x$ = number of hutches, when you answer the following questions.

(a)  
(i) Determine the linear equation that represents the monthly cost for this company.

\[ C = 60x + 5500 \]

(ii) The company sells hutches to a business that will buy up to 25 hutches every month, at $260 each.

Determine the linear equation that represents the monthly sales revenue for this company from sales to this business.

\[ R_s = 260x \]

The company also sells hutches to a second business for $180.00 each.

Assuming the first business buys 25 huches per month, then the linear equation for the total sales revenue becomes:

\[ R_2 = 180x + 2000 \text{ for } x > 25 \text{ hutches} \]

(b)  
(i) Graph the three equations on the axis below and indicate the break-even point for the company.

Question 2 continues.
Question 2 (continued)

(ii) Algebraically determine the minimum number of rabbit hutchs that need to be made and sold for the company to make a profit.

\[
\begin{align*}
\text{Break Even: } \text{Revenue} &= \text{Cost} \\
180x + 2000 &= 60x + 5500 \\
180x - 60x &= 5500 - 2000 \\
120x &= 3500 \\
x &= \frac{3500}{120} \\
\therefore x &= 29.17
\end{align*}
\]

\therefore \text{Minimum number of hutchs is } 30

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5500</td>
<td>7900</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_l</td>
<td>0</td>
<td>6500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>25</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_l</td>
<td>6500</td>
<td>9200</td>
</tr>
</tbody>
</table>
Question 3 (Approximately 12 minutes)

The table below shows the mass of salt (g) that can dissolve in 100 mL of water, at different temperatures (°C).

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Mass of salt (g) S</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>59.5</td>
</tr>
<tr>
<td>20</td>
<td>65.2</td>
</tr>
<tr>
<td>30</td>
<td>70.6</td>
</tr>
<tr>
<td>40</td>
<td>75.5</td>
</tr>
<tr>
<td>50</td>
<td>80.2</td>
</tr>
<tr>
<td>60</td>
<td>85.5</td>
</tr>
<tr>
<td>70</td>
<td>90.0</td>
</tr>
</tbody>
</table>

(a) Determine the **exponential and linear** models that fit this data, giving coefficients to three decimal places.

Exponential Model: \( S = 56.716 e^{0.007T} \)

Linear Model: \( S = 0.506T + 54.971 \)

(b) Give two appropriate reasons the **linear** model should be chosen over the exponential model to estimate the mass of salt that will dissolve at a given temperature.

Possible reasons...

- No obvious pattern in residuals
- \( r^2 \) value close to 1.0
- A simpler model to use.
- Maybe more reliable at higher temperatures (extreme values)

Question 3 continues.
Question 3 (continued)

(c) State the correlation coefficient for the linear model.

Explain what this value means in terms of the relationship between S and T.

\[ r = 0.9995 \]

There is a strong positive correlation between temperature and amount of salt dissolved. The greater the temperature, the greater the mass of salt dissolved.

(d) Using the linear model, predict the mass of salt that will dissolve in 100 mL of water at 25°C.

\[ S_{(25)} = 67.6 \text{ g} \]

(e) Using the linear model, determine algebraically the predicted minimum temperature required to dissolve 93.0 g of salt in 100 mL of water.

\[
\begin{align*}
93.0 &= 0.506T + 54.971 \\
93.0 - 54.971 &= 0.506T \\
T &= 75.12 \degree C
\end{align*}
\]

(f) Comment on the reliability of your predictions in parts (d) and (e).

The linear model has an \( r^2 \) value of 0.9989.

The reliability of the prediction in (d) is reliable due to interpolation of a model with a high \( r^2 \).

The reliability of the prediction in (e) is much less than (d) due to extrapolation of the model.
**Question 4** (Approximately 8 minutes)

The distance $D$, in metres, that an object slides along a smooth surface in $t$ seconds can be modelled by the equation:

$$D = 20t - 0.125t^2$$

(a) (i) Write the equation for the speed $\frac{dD}{dt}$.

$$\frac{dD}{dt} = 20 - 0.250t$$

(ii) State the units of $\frac{dD}{dt}$.

$m/s$

(b) Use your speed equation to determine the speed of the object when $t = 10$ seconds.

$$\text{Speed} = 20 - 0.250(10)$$

$$= 17.5 \text{ m/s}$$

(c) Sketch the graph of speed against time as the object slides, over the first 40 seconds.
**Question 5** (Approximately 8 minutes)

A metal frame for a box is to be constructed from a piece of wire of length 310 cm.

The box has to have a base length twice the width $x$, as shown in the diagram below.

(a) Show that the volume $V$ (cm$^3$) of the box is:

\[ V = -6x^3 + 155x^2 \]

\[ \begin{align*}
V &= l \cdot w \cdot h \\
 &= (2x)(x)(h) \\
\therefore V &= 2x^2h \\
\end{align*} \]

Using the given conditions:

\[ 4(2x) + 4(x) + 4(h) = 310 \]

\[ 12x + 4h = 310 \]

\[ 4h = 310 - 12x \]

\[ \therefore h = \frac{155}{x} - 3x \]

\[ \begin{align*}
\text{Sub (i) into (iv)} \quad & V = 2x^2 \left( \frac{155}{x} - 3x \right) \\
& = 155x^2 - 6x^3 \\
\therefore V &= -6x^3 + 155x^2 \\
\end{align*} \]

\[ \text{as required.} \]

Question 5 continues.
Question 5 (continued)

(b) Use calculus techniques to determine the maximum volume of the box.

\[ \frac{dV}{dx} = -18x^2 + 310x \]

\[ = -2x(9x - 155) \]

When \(-2x(9x - 155) = 0\),

\[ x = 0 \quad \text{or} \quad 9x - 155 = 0 \]

\[ x = 0 \quad \text{or} \quad 9x = 155 \]

\[ x = 155/9 \]

\[ V\left(\frac{155}{9}\right) = 6 \left(\frac{155}{9}\right)^3 + 155 \left(\frac{155}{9}\right)^2 \]

\[ = 15324.6 \]

\[ \therefore \text{Max Volume is } 15324.6 \text{ cm}^3 \]
Question 6 (Approximately 12 minutes)

The flight path of a golf ball could be modelled using the formula

\[ H = \frac{-x^2(x - 240)}{40000} = \frac{-x^3}{40000} + \frac{3x^2}{500}, \]

where \( H \) is the height of the ball (m) and \( x \) is the horizontal distance travelled (m).

(a) A 15 m high tree is at a horizontal distance of 60 m from the start of the path of the ball. Show that according to this model a ball aimed at the tree would go over the top of the tree.

\[ H(60) = \frac{-60^2(60 - 240)}{40000} \]

\[ H(60) = 16.2 \text{ m} \quad \therefore \text{ The ball will go over the tree} \]

(b) Use calculus techniques to show that the maximum height the ball reaches in flight is 51.2 m.

Max. height is when \( \frac{dH}{dx} = 0 \)

\[ \frac{dH}{dx} = \frac{-3x^2 + \frac{3x}{500}}{40000} \]

\[ = \frac{-3x^2 + 480x}{40000} \]

\[ = \frac{-3x}{40000}(x - 160) \]

When \( \frac{-3x}{40000}(x - 160) = 0 \)

\[ x = 0 \quad \text{or} \quad x = 160 \]

\[ \therefore \text{Max. Height} = \frac{-160^2(160 - 240)}{40000} \]

Max Height = 51.2 m.

Question 6 continues.
Question 6 (continued)

(c) Sketch a graph of the flight path of the ball for horizontal distances from 0 m to 240 m. Show intercepts and turning points.
**Question 7** (Approximately 8 minutes)

In a particular computer game, a character’s **Power (P)** is the product of its **Knowledge (K)** and **Luck (L)**.

For each character level gained, the character’s Knowledge increases by two points and its Luck decreases by two points.

(a) Complete the table.

<table>
<thead>
<tr>
<th>Character Level (x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge (K)</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>2x</td>
</tr>
<tr>
<td>Luck (L)</td>
<td>100</td>
<td>98</td>
<td>96</td>
<td>94</td>
<td>100 - 2x</td>
</tr>
<tr>
<td>Power (P)</td>
<td>0</td>
<td>196</td>
<td>384</td>
<td>564</td>
<td>2x(100-2x)</td>
</tr>
</tbody>
</table>

(b)  

(i) Show that the Power P can be modelled using the equation

\[ P = 200x - 4x^3, \]

where \( x \) is the character level.

\[
P = 2x(100 - 2x)
\]

\[
P = 200x - 4x^3 \quad \text{as required}
\]

(ii) The maximum character level occurs when the Power P is a maximum. **Use calculus techniques** to determine the maximum character level.

Max Power when \( \frac{dP}{dx} = 0 \)

\[ \frac{dP}{dx} = 200 - 8x \]

when \( 200 - 8x = 0 \)

\[ x = 25 \]

... Maximum character level is 25...
Question 8 (Approximately 7 minutes)

A flight from Rome (41.9°N, 12.5°E) to Dubai (25.0°N, 55.3°E) was scheduled to depart at 2:10 pm on Tuesday.

(a) What time would this be in Dubai?

\[
\text{Rome : } 12.5°E \rightarrow 12.5 + 0.83 \text{ hours GMT} = 13.33\text{ hours GMT}
\]

\[
\text{Dubai : } 55.3°E \rightarrow 55.3 + 3.69 \text{ hours GMT} = 15.09\text{ hours GMT}
\]

\[
\therefore \text{Dubai is } 3\text{ hours ahead of Rome.}
\]

\[
\text{The flight leaves at } 5:10 \text{ pm on Tuesday.}
\]

(b) The flight departed Rome on time and arrived in Dubai at 12:15 am on Wednesday.

Determine the average speed in km/h of the plane during the flight, assuming it followed the shortest possible route.

\[
\cos \theta = \sin (41.9°) \times \sin (25.0°) + \cos (41.9°) \times \cos (25.0°) \times \cos (32.1°)
\]

\[
= 0.7772
\]

\[
\theta = \cos^{-1}(0.7772)
\]

\[
\theta = 39.0°
\]

\[
D = \frac{2 \times \pi \times \theta}{360}
\]

\[
= \frac{2 \times 3.14 \times 39}{360}
\]

\[
D = 4336.6 \text{ km}
\]

\[
\text{Speed} = \frac{\text{Distance}}{\text{Time}}
\]

\[
= \frac{4336.6 \text{ km}}{7.08 \text{ hrs}}
\]

\[
\therefore \text{Speed} = 612.3 \text{ km/h}
\]
Question 9 (Approximately 7 minutes)

A plane (A) is directly above a ship (C) as shown in the diagram below. The plane is 270 km away from a port (B) which is on the horizon when viewed from the plane.

(a) Calculate the altitude of the plane when it is directly above the ship.

\[ a = R + x \]

\[ a^2 = 6371^2 + 270^2 \]

\[ x = 5.7 \] km

Altitude 5.7 km

\[ a = 6376.7 \text{ km} \]

(b) Determine the angle of separation (θ) of the ship and the port.

\[ \tan \theta = \frac{270}{6371} \]

\[ \theta = \tan^{-1} \left( \frac{270}{6371} \right) \]

\[ \theta = 2.2^\circ \]

(c) Determine the time it would take the ship to reach the port at an average speed of 10 knots.

\[ D = 60 \times \theta \]

\[ D = 145.8 \text{ n miles} \]

\[ \text{Time} = \frac{\text{Distance}}{\text{Speed}} \]

\[ \text{Time} = 14 \text{ hrs and 35 mins} \]

or 14.58 hours
Question 10 (Approximately 10 minutes)

A light aircraft is scheduled to fly directly from Wagga Wagga to Sydney. Due to bad weather, the aircraft must fly another route.

(a) The first leg of the journey has the aircraft travelling due north from Wagga Wagga (35°07'S, 147°22'E) to Point A (33°52'S, 147°22'E).

Determine the distance from Wagga Wagga to Point A, to the nearest km.

\[ D = \frac{2\pi R}{360} \times 360 \]

\[ = 2 \times \pi \times 63.71 \times \frac{125}{360} \]

\[ \therefore D = 139 \text{ km} \]

(b) The second leg of the journey has the aircraft travelling due east from Point A (33°52'S, 147°22'E) to Sydney (33°52'S, 151°12'E).

Determine the distance from Point A to Sydney, to the nearest km.

\[ D = \frac{2\pi R}{360} \cos^2 \left( \frac{360}{2} \right) \]

\[ = 2 \times \pi \times 63.71 \times \cos \left( \frac{33^\circ52'}{2} \right) \times \frac{\cos \left( \frac{33^\circ52'}{2} \right)}{360} \]

\[ \therefore D = 354 \text{ km} \]

(c) The flight normally takes 90 minutes.

If the aircraft has an average speed of 305 km/h, determine the number of minutes the aircraft is late into Sydney.

\[ \text{Distance} = 139 + 354 = 493 \text{ km} \]

\[ \text{Speed} = 305 \text{ km/h} \]

\[ \text{Time} = \frac{\text{Distance}}{\text{Speed}} \]

\[ = \frac{493}{305} \]

\[ \therefore \text{Time} = 1 \text{ hr } 37 \text{ mins} \]
Question 11 (Approximately 6 minutes)

The diagram above shows a flat section of land with a 45 m high tower. Point A is directly east of the tower.

(a) The angle of elevation from Point A to the top of the tower is 40°.

Determine the horizontal distance from Point A to the base of the tower (T).

\[ \overrightarrow{AT} = \frac{45}{\tan(40°)} \]

\[ \overrightarrow{AT} = 53.16 \text{ m} \]

Question 11 continues.
Question 11 (continued)

(b) Point P is 30 m directly south of the tower. Point P is in a direct line of sight of Point A when viewed from Point B.

Determine the bearing of Point A from Point B.

\[ \tan \theta = \frac{53.6}{30} \]

\[ \theta = \tan^{-1} \left( \frac{53.6}{30} \right) \]

\[ \theta \approx 60.46^\circ \]

\[ \text{ Bearing of A from B is N} 60^\circ 46' \text{ E} \]
Question 12 (Approximately 6 minutes)

The diagram of a steel construction is shown below.

\[ \triangle \text{BAD} : \]
\[
\cos \theta = \frac{2^2 + 4.5^2 - 5.6^2}{2 \times 2 \times 4.5}
\]
\[
\cos \theta = \frac{-0.395}{-0.395}
\]

\[ \triangle \text{ACD} : \]
\[
\overline{CD}^2 = 5^2 + 4.5^2 - 2 \times 5 \times 4.5 \times -0.395
\]
\[
\overline{CD}^2 = 6.31025
\]
\[
\overline{CD} = 7.94
\]

The length of the beam CD is 7.94 m.
Question 13 (Approximately 6 minutes)

The speed of 200 cars travelling around a bend was recorded and the results summarised in the cumulative frequency polygon (ogive) below.

(a) The recommended speed for this bend is 60 km/h. How many of the cars were travelling at less than the recommended speed?

64 cars.

t C

(b) The actual speed limit for this section of road is 90 km/h. State the percentage of the cars recorded travelling above the speed limit.

14% above speed limit.

t C

c) What is the interquartile range (IQR) of this data? Explain what this value means.

\[ IQR = Q_3 - Q_1 \]

\[ = 82 - 56 \]

\[ IQR = 26 \text{ km/h} \]

The middle 50% of cars were travelling within 26 km/h of each other.

For Marker
Use
Only

Page 3
Question 14 (Approximately 10 minutes)

The back-to-back stem and leaf plot below shows the average daily wind speeds at two possible wind farm sites, Site A and Site B, over a 15-day period.

<table>
<thead>
<tr>
<th>Site A</th>
<th>Site B</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 9 7 5 3 3</td>
<td>0 5 5 6 6 7 8 8 9</td>
</tr>
<tr>
<td>9 7 6 5 5 3</td>
<td>1 0 2 4 5 5</td>
</tr>
<tr>
<td>7 3 0</td>
<td>2 5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Key: 1|4 = 14 km/h

(a) Determine five-figure summaries for Site A and for Site B.

Site A: 3 7 15 19 37

Site B: 5 6 9 15 34

(b) Draw below side-by-side box and whisker plots for Site A and Site B.

For Marker Use Only

Question 14 continues.
(c) (i) Using statistical information gained from parts (a) and (b), describe how the average daily wind speeds at these two sites differ.

Site A has a range of 24 km/h and 18% of 12 km/h, which are both lower than Site B, range of 28 km/h and 18% of 9 km/h.
Site A has a lower min. and max. speed than Site B.
Site B has more consistent wind speeds than Site A.

(ii) Give an implication of the data for someone deciding whether a wind farm should be built at Site A or at Site B.

- If consistency of wind speed is required, then Site B would be best.

- If higher wind speeds are required then Site A is best.

- 50% of wind speed is above 15 km/h for Site A whereas only 25% of wind speed for Site B.
- Median wind speed for Site A is 15 km/h whereas it is only 9 km/h for Site B.
Question 15 (approximately 12 minutes)

A random sample of 100 students trialled a national literacy test. The marks for this national test are expected to approximate a normal distribution.

The results of these 100 students were used to estimate the mean and standard deviation of the population of student marks.

Estimated Population Mean Mark ($\mu$): 55.4
Estimated Population Standard Deviation ($\sigma$): 9.30

(a) The ‘pass mark’ for the test was initially set at 45.

Determine the percentage of the national student population who would be expected to pass the test.

\[ P(x > 45) = \text{normcdf}(45, \infty, 9.30, 55.4) \]
\[ = 0.8683 \]
\[ \therefore 86.83\% \text{ would be expected to pass.} \]

(b) One year, 9600 students undertook the test: 350 students did not pass.

Determine the pass mark for that year to the nearest whole number.

\[ \% \text{ Did not pass} = \frac{350}{9600} \times 100 \]
\[ = 3.645\% \]
\[ x = \text{invnormcdf}(0.03645, 9.30, 55.4) \]
\[ \therefore x = 38.72 \]
\[ \therefore \text{Pass mark of 39.} \]
Question 15 (continued)

(c) Use the estimated population data to circle whichever of the following is most likely to be the mean mark ($\bar{x}$) and standard deviation ($\delta_x$) of the original sample of 100 students.

A. $\bar{x}: 55.4$  
$\delta_x: 9.30$

B. $\bar{x}: 55.4$  
$\delta_x: 9.34$

C. $\bar{x}: 55.4$  
$\delta_x: 9.25$

D. $\bar{x}: 54.8$  
$\delta_x: 9.30$

E. $\bar{x}: 56.0$  
$\delta_x: 9.30$
**Question 16** (Approximately 8 minutes)

A machine is supposed to deliver material in 45.00 g lots. A sample of twenty random lots was taken from this machine, and the mean and standard deviation of this sample were determined to be 44.85 g and 0.31 g respectively.

Assuming the distribution is normal, use an appropriate t-test to decide whether the machine is delivering, on average, lots of less than 45.00 g.

(a) State both the Null and Alternative Hypotheses.

\[
H_0: \mu = 45.00 \text{ g per packet} \\
H_1: \mu < 45.00 \text{ g per packet}
\]

(b) State the results of the t-test.

\[
\bar{x} = 44.85 \text{ g}, \quad p = 0.021708
\]

(c) State your conclusion.

If the machine is delivering 45.00 g per packet, then the probability of twenty samples with a mean of 44.85 g is 2.17%.

As \( p < 0.05 \), we reject \( H_0 \) and conclude that the machine is delivering, on average, lots of less than 45.00 g.
Question 17 (Approximately 8 minutes)

(a) Use appropriate formulas to calculate the interest accrued over 60 days in an account with an opening balance of $16,750, when the account pays:

(i) a flat interest rate of 1.3% p.a., over the 60 days.

\[
I = PRT
\]

\[
= 16,750 \times 0.013 \times \frac{60}{365}
\]

\[
I = 35.79
\]

(ii) an interest rate of 1.3% p.a., compounding daily over the 60 days.

\[
A = P \left(1 + \frac{r}{n}\right)^n
\]

\[
= 16,750 \left(1 + \frac{0.013}{365}\right)^{60}
\]

\[
A = 16,785.83
\]

\[
I = 35.83
\]

(b) An account offers an interest rate of 6.40% p.a., compounding weekly.

Convert this to an effective interest rate.

\[
E = \left(1 + \frac{r}{n}\right)^n - 1
\]

\[
= \left(1 + \frac{0.064}{52}\right)^{52} - 1
\]

\[
E = 0.0661
\]

\[
\text{Effective Interest Rate} = 6.61\% \text{ p.a.}
\]
Question 18 (Approximately 9 minutes)

Chloe invests $400 per month in a superannuation account for 20 years. This investment accrues interest at 5.7% p.a., compounding monthly.

(a) **Using an appropriate formula**, determine the expected superannuation payout at the end of the 20 years.

\[
F = \frac{R \cdot (1+i)^t - 1}{i} \cdot \frac{1}{(1+i)^t - 1}
\]

\[
= \frac{400 \cdot (1 + 0.057)^{240} - 1} {0.057} \cdot \frac{1}{(1+0.057)^{240} - 1}
\]

\[
= \$179,233.92
\]

(b) Suppose that Chloe had added an extra one-off investment of $50,000 into this superannuation account at the end of the sixth year.

(i) Determine her resultant superannuation payout after 20 years.

Treat one-off investment as compound interest... Formula question:

\[
A = 50,000 \cdot (1 + 0.057)^{168} \quad i = 14 \times 20
\]

\[
\therefore A = \$110,845.10
\]

Payout = $110,845.10 + $179,233.92

= $290,079.02

(ii) Calculate the amount of interest that Chloe would then be paid over the 20-year period.

\[
I = 290,079.02 - (400 \times 240 + 50000)
\]

\[
\therefore I = \$144,079.02
\]
Question 19 (Approximately 10 minutes)

George recently purchased a new machine for his business for $76 000. He is investigating two choices of depreciation method for tax purposes.

(a) The machine may be depreciated using a reducing balance method of 27% p.a. Determine the book value of this machine after four years using this method.

\[ A = P(1 - i)^n \]

\[ = 76 000 (1 - 0.27)^4 \]

\[ A = $21 582.66 \]

Book value is $21 582.66

(b) The machine may be depreciated using a straight line method of 20% of the purchase price each year. Determine the book value of this machine after four years using this method.

\[ V = 76 000 - (4 \times 0.20 \times 76 000) \]

\[ V = $15 200 \]

Book value is $15 200

(c) Discuss when George should depreciate this new machine using a reducing balance method and when it would be better for him to use the straight line method.

Make sure you take into account the book values over five years for each model.

The lower book value is best for tax purposes because a greater depreciation value is able to be claimed. The two models produce the same book value at 3.14 years.

The reducing balance model produces a lower book value in years 1 to 3 whereas the straight line model produces a lower book value in years 4 and 5.

: Depreciating the item up to 3 years choose reducing balance otherwise choose straight line for 4 to 5 years.
Question 20 (Approximately 9 minutes)

Katie takes out a loan of $38,700 for a new car, to be repaid over five years. The interest rate on her loan is 10.75% p.a., compounding monthly.

(a) Determine her **fortnightly** repayment amount.

Using TVM

\[ N = 130 \]

\[ I\% = 10.75 \]

\[ PV = 38700 \]

\[ \text{pay } \$385.20 \text{ per fortnight} \]

\[ FV = 0 \]

\[ P/I = 26 \]

\[ C/Y = 12 \]

\[ PMT = END \]

(b) If Katie chooses to pay $450 each **fortnight** instead, how many fortights will it take her to pay out the loan?

Using TVM

\[ * N = 106.43 \]

\[ I\% = 10.75 \]

\[ PV = 38700 \]

\[ \text{payments at } \$450 \]

\[ PMT = -450 \]

\[ FV = 0 \]

\[ P/I = 26 \]

\[ C/Y = 12 \]

\[ PMT = END \]

So it will take 107 fortights.
# Award Distribution

<table>
<thead>
<tr>
<th></th>
<th>EA</th>
<th>HA</th>
<th>CA</th>
<th>SA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>This year</td>
<td>11% (163)</td>
<td>16% (233)</td>
<td>36% (538)</td>
<td>37% (541)</td>
<td>1475</td>
</tr>
<tr>
<td>Last year</td>
<td>9% (127)</td>
<td>16% (215)</td>
<td>37% (508)</td>
<td>38% (529)</td>
<td>1379</td>
</tr>
<tr>
<td>Last year (all examined subjects)</td>
<td>10 %</td>
<td>19 %</td>
<td>39 %</td>
<td>32 %</td>
<td></td>
</tr>
<tr>
<td>Previous 5 years</td>
<td>11 %</td>
<td>19 %</td>
<td>37 %</td>
<td>33 %</td>
<td></td>
</tr>
<tr>
<td>Previous 5 years (all examined subjects)</td>
<td>11 %</td>
<td>19 %</td>
<td>39 %</td>
<td>30 %</td>
<td></td>
</tr>
</tbody>
</table>

# Student Distribution (SA or better)

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Year 11</th>
<th>Year 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>This year</td>
<td>50% (742)</td>
<td>50% (733)</td>
<td>64% (937)</td>
<td>36% (538)</td>
</tr>
<tr>
<td>Last year</td>
<td>47% (642)</td>
<td>53% (737)</td>
<td>65% (890)</td>
<td>35% (489)</td>
</tr>
<tr>
<td>Previous 5 years</td>
<td>47%</td>
<td>53%</td>
<td>58%</td>
<td>42%</td>
</tr>
</tbody>
</table>