General Mathematics
Course Code: MTG315115

Part 1 - Bivariate Data

Examiners found that the main areas that candidates and teachers need to address are the reading and interpretation of the questions asked. General communication needs to be improved, with the expected inclusion of units with all answers and use of appropriate number of decimal places. They should also consider the reasonableness of their answers and make appropriate corrections.

Candidate’s answers to the worded questions need to include justifying choices and selections. Also candidates should be encouraged to make concise worded answers. Written material should focus on answering the question and not repeating elements of the question. It should be noted that for three and four mark questions candidates should give three of four valid and separate points for full marks. For full marks detailed answers displaying understanding was required and not just facts.

Candidates must also provide some Mathematical ‘substance’ in questions worth 3 or more marks. Candidates are reminded that rulers are a necessary part of their stationary equipment in this subject and appropriate use of a ruler is encouraged.

Spare graphs are provided at the end of the paper and candidates need to be encouraged that it is more appropriate for them to use these when errors have occurred rather than crossing out and writing over the top. Across the questions candidates were inconsistent in their use of significant figures or failed to round correctly. Many candidates lost some marks for poor rounding or failing to give the required number of decimal places.

Question 1
(a) This question was very well done by most candidates.
(b) (i) This question was quite well done by most candidates. The majority did this well but there was a significant number that calculated percentages of the row totals rather than the column totals. The column graph was done quite well with common errors being lack of labels for the key and columns. For the purpose of comparison it is important that the variables were in the same order for the columns.
(ii) This question was quite well done by most candidates. Candidates were able to describe the findings quite well but often did not state preferences or make comparisons. It should be noted that percentages should be compared not numbers. Candidates did not receive full marks without reference to the possible association between the variables and very few did this.

Question 2
a) This question was very well done by most candidates.
b) This question was quite well done by most candidates. Common errors included using x and y rather than T and D which lead to further errors in the question and incorrect rounding and incorrect number of decimal places.
c) This question was quite well done by most candidates. Many candidates gave the strong, negative but did not give the full interpretation of the correlation value naming the variables in the final statement. A few candidates incorrectly gave the $r^2$ value.
d) This question was quite well done by most candidates. To get full marks for this question at least one line of algebraic manipulation must be shown. Other common errors included lack of units and not including the x1000 km in final answer.
e) This question was quite well done by most candidates. Most candidates recognised that the answer was based on interpolation and therefore reliable but did not justify the appropriateness of the model using the $r^2$ value and the size of the data sample.
f) This question was reasonably well done by most candidates. Most candidates were able to produce a residual plot. Common errors were lack or poor choice of scale leading to plotting errors. Some ignored the x scale and just showed the pattern produced from their calculator. A good tip here is to try and use the full grid area.
g) This question was reasonably well done by most candidates. Some candidates failed to recognise that this was NOT an appropriate model due to the clear pattern in the residuals, despite the high r and r² values. Some used the recipe response to residual analysis with very little connection to this model. Candidates wrongly identified outliers when there were none. A good tip is to look at the original graph to assist their interpretation of whether the model is appropriate. Describing the pattern seen is important rather than stating even or uneven distributions.

Question 3
(a) This question was very well done by most candidates. Some candidates failed to plot the raw data or plotted it in the incorrect place.
(b) This question was very well done by most candidates. A good tip is to give the same number of decimal places as the table provides.
(c) This question was very well done by most candidates. Some candidates failed to plot the raw data or plotted it in the incorrect place. A good tip is to give the same number of decimal places as the table provides.
(d) This question was quite well done by most candidates. Most candidates recognised that deseasonalising the data had the effect of smoothing and that there was a long term downward trend. However since this was a three mark question some evidence of further understanding was required.
(e) This question was reasonably well done by most candidates. Although candidates regularly identified the fourth quarter as the highest water usage on a deseasonalised basis, the explanations given generally showed little or no understanding of what this means.
(f) This question was quite poorly done by most candidates. Many candidates could not identify the correct quarter number. (x=17)

Most were able to find the deseasonalised value but failed to recognise that this was not the final answer. There were quite a few candidates that then divided by the index rather than multiplying. There was a penalty for lack of units or incorrect units here.

Part 2 – Growth and Decay in Sequences

Examiners found that the main area that candidates and teachers need to address is the development of skills required to solve algebraic equations. Many candidates either ignored the instruction to show solutions algebraically or were not able to display this skill.

Graphing skills also need addressing. Candidates often found scaling a challenge and mistakenly used a trend line where discrete points were required.

Candidates’ communication was generally good and many candidates showed a sound understanding of this topic. Candidates are encouraged to be more familiar with their calculator. They should pay more attention to starting values in tables and ensure consistent use of appropriate variables.

Question 4
(a) This question was very well done by most candidates.
(b) This question was quite well done by most candidates. Errors were made by candidates when they attempted to simplify the rule, which was not required for this exam, but could be asked. The skill of simplifying should be practised more often prior to the exam. Some candidates missed the addition sign in the rule.
(c) This question was quite well done by most candidates. Candidates’ algebraic skills need to be improved – some algebraic manipulation was often omitted. Candidates often substituted rather than solving, with arithmetic solutions given rather than algebraic solutions.
(d) This question was very well done by most candidates.
Question 5
(a) This question was very well done by most candidates.
(b) This question was quite well done by most candidates. Candidates’ algebraic skills often let them down in this question. Quite a few candidates showed a lack of understanding of indices, often stating that \( r^2 = r \times 2 \) and \( r^0 = 1 \).
(c) This question was reasonably well done by most candidates. Common errors revolved around calculator use and data entry. Some candidates solved for the sum rather than the term. Often the answer was left as 9.28 or incorrectly rounding down to the 9th year rather than stating the 10th year.

Question 6
(a) This question was very well done by most candidates.
(b) This question was very well done by most candidates.
(c) This question was quite well done by most candidates. Some candidates solved using the term rather than the sum. The explanation or justification of appropriated model was often omitted.

Question 7
(a) This question was very well done by most candidates.
(b) This question was quite poorly done by most candidates. Common errors included starting the graph at \( n=1 \) rather than at \( n=0 \) and using a trend line instead of discrete points. Accuracy of points could be improved.
(c) This question was quite poorly done by most candidates. There was an issue that candidates needed to use the same set of axes for this part as well as part (b) and hence the dependant variable axis was poorly scaled to suit both parts. Candidates often forgot to label or use a key to distinguish between the two graphs. The equation constructed often had \( W_0 \) missing. The descriptions of how the wallaby numbers changed was usually well done, except that some candidates mentioned an exponential decrease.
(d) This question was very well done by most candidates. The only issue was that some candidates did not make their final answer clear.

Part 3 – Finance
Candidates were quite poorly prepared for the examination of this section. Many candidates had trouble getting started in a range of questions.

Communication was generally poor in this section, where currency was often quoted as $0.7 instead of $0.70, for example. The $ was often left out of currency as well.

Examiners found that the main areas that candidates and teachers need to address is the development of an understanding of real life examples in finance, extracting relevant information out of the question and strategies required to solve these problems.

Candidates should be encouraged to leave fractions in formula rather than converting to a decimal and rounding. For example, \( i \) in compound interest and annuity formula.

It should be stressed that candidates spend the time reading the exam paper before diving in and a much more rigorous practise of past exam questions is required.

Question 8
(a) This question was poorly done by most candidates. Common errors included this use of compound interest, the confusion of 24 or 25 days and the use of \( \frac{3.65}{365} \) rather than \( \frac{3.65}{100} \) as a means of calculating percentage. The setting out of this question was very poor.
(b) This question was done very well by most candidates.
Question 9
(a) This question was quite well done by most candidates. Candidates often misread 4.40% as 4%. Candidates were penalised if they were not able to solve for \( P \) algebraically and used the TVM instead. Some used the annuity formula instead of the compound interest formula.
(b) This question was done very well by most candidates.
(c) This first part of the question was done quite well by most candidates. They were able to calculate the new price of the car but many did not find the amount extra required to buy the car.

Question 10
(a) This question was very poorly done by most candidates. Many did not show sufficient understanding of whether this was an annuity in advance or arrears question. About half of the candidates attempted to use a formula to solve this question, even though \( p/y \) and \( c/y \) were different and therefore the TVM should always be used instead. Those candidates who did chose to use the TVM and selected BEGIN then used the incorrect value for \( N \).
(b) This question was very well done by most candidates. Some creative approaches were used to solve the problem.
(c) This question was quite well done by most candidates. Many candidates realised that there were only 12 years that the scholarship could be awarded at $5 000. Some appeared to not read the question or were confused about the word annual.

Question 11
(a) This question quite poorly done by most candidates. Many candidates were unsure about what to do in this question and attempted to use a compound interest formula. Candidates often neglected to finish the question.
(b) This question was reasonably well done my most candidates. The common error was that many candidates did not understand what was required to show that Bill owes $10 741. Most candidates chose the correct formula but mistakes were made with substitution.
(c) This question was quite well done by most candidates. Some candidates did not complete this question and errors were made in converting percentages to decimals.
(d) This question was poorly done by most candidates, with many unsure on what they were trying to find. The time saved was often rounded incorrectly and numerous candidates found it difficult to convert months to years and months.

Part 4 - Trigonometry
Examiners found that the main areas that candidates and teachers need to address is in the area of communication, the general reading of the question and construction of diagrams using bearings. All candidates need to read the questions with care and answer the questions asked in particular number of decimal places. They should also consider the reasonableness of their answers and make appropriate corrections. General communication needs to be improved, with the expected inclusion of units with all answers and use of appropriate number of decimal places.

As marks are now used in the exam then it is essential to show clear working to obtain full marks in any question worth 3 or more marks.

Check calculator is in degree mode as some candidates had correct working but incorrect answers. Diagrams need to be legible and clearly show the angles and lengths.

Candidates are advised to take the time to understand what the question is asking before commencing.

Question 12
(a) This question was very well done by most candidates. However some candidates did not use the small circle formulae, and confused the angles \( \theta \) and \( \alpha \).
(b) This question was reasonably well done by most candidates. Clear working was not always shown and marks were deducted if only answers were given. Candidates confused hours and minutes with decimal hours giving incorrect arrival time as they tried to add decimals with minutes. There was also some
confusion between how to write hours and minutes and some candidates were writing as degrees and
seconds.

Question 13
(a) This question was very well done by most candidates. Nearly all candidates used Heron’s formula correctly
but some candidates divided by 3 instead of 2 when finding S. Candidates need to note they should not
round 7.55 to 7.5. Some candidates did not give answer to 2 decimal places as this was asked for in
question. Some candidates did not use correct units of m² for the area
(b) This question was reasonably well done by most candidates. Some candidate did not recognise that large
triangle was found by using \( A = \frac{\text{base} \times \text{height}}{2} \) and consequently used 6 m as an edge length or made the
question considerably harder by finding an angle using cosine rule then alternative area formula. Some
candidates failed to include appropriate units. Some candidates did not finish answering the question and
did not find the shaded area.

Question 14
(a) This question was very well done by most candidates. Some candidates failed to answer the question and
did not write the standard time difference was 14 hours.
(b) This question was very well done by most candidates. Some candidates failed to include the date or
wrongly subtracted the time difference.
(c) This question was quite poorly done by most candidates. Many candidates failed to comment on the
shortest route as a Great Circle route. Many candidates did not note the 2 dimensional nature of a map
compared to the spherical nature of the earth. Many answers were considered as showing no
understanding of the question asked.
(d) This question was reasonably well done by most candidates. Most candidates knew to use the angular
separation formula first however there was a number of candidates who found the longitude difference
incorrectly or used the longitude instead of latitude in the formula. Some candidates could not find the
angle correctly. Most candidates used the correct formula to find the distance in km but some candidates
are still confused when to use which formula.
(e) This question was very poorly done by most candidates. Candidates were not able to clearly communicate
a solution and there were many errors in finding the correct travel time. Incorrect adjustment for zone time
difference was common. Often a time was just given with no clear indication of how it was found. Very
few candidates actually found the correct speed but most correctly used the formula with their values. It is
important to note answers of 5 km/h is not an appropriate answer for the speed of the plane.

Question 15
(a) This question was very well done by most candidates. There was often insufficient evidence for calculating
the required answer and putting it on the diagram was not enough. Some candidates did not clearly show
the 140° angle and some left it off altogether.
(b) This question was reasonably well done by most candidates. Some candidates failed to give the answer to
1 decimal place. Some candidates wrongly used the Sine rule with inappropriate angles. Some candidates
correctly substituted into the cosine formula but could not find the correct length. Check in correct mode
(degree).
(c) Finding the angle was quite well done by most candidates. Some candidates failed to convert the correct
answer to degrees and minutes. However many candidates could not find the correct bearing. Candidates
were unable to select the correct angles to be used in order to calculate the true bearing angle.

Part 5 - Graphs and Networks
Examiners found that the main areas that candidates and teachers need to address is to ensure that all
candidates have read the questions with care and answer the questions asked. Candidates need to make sure
that they answer every question on a section – where there is a mark make sure you have given an answer.
Make sure candidates fill in every box/circle when asked to. Many missed marks by not fully completing diagrams.
Generally most candidates gave units where required.

Giving the examiner clear guidance in what you are doing is very beneficial. They should also consider the
reasonableness of their answers and make appropriate corrections.
Spare graphs are provided at the end of the paper and candidates need to be encouraged that it is more appropriate for them to use these when errors have occurred rather than crossing out and writing over the top.

The concept of cuts appeared to be poorly understood by many candidates. Most could calculate basic cuts and give maximum flow but their explanation as to why or what to upgrade revealed that they lacked a deeper understanding. In particular that a cut (with the exception of the minimum cut) has no basis in reality in terms of inflow and outflow. As far as cuts and maximum flow are concerned the capacity of an edge is not affected by the capacity of a preceding edge.

If two cuts share a common edge then changing the value of that edge changes the value of both cuts, candidates need to both state this and identify the next lowest unaffected cut.

If dummy edges, Hungarian algorithm and drawing an activity diagram for critical path analysis were included in the exam perhaps more examples of misunderstanding would have been highlighted.

**Question 16**
(a) This question was very well done by most candidates.
(b) This question was quite poorly done by most candidates. Many candidates did not calculate cut IV correctly – they missed the fact that there was a reverse flow. It was quite common for a candidate to not select the minimum cut to find the maximum flow.
(c) This question was very poorly done by most candidates. Many candidates were unable to answer this question completely. Most candidates lost marks by not stating that cuts II and III were affected by changes to CF and did not recognise that if the next minimum cut was also upgraded they needed to look for the next one.

**Question 17**
(a) This question was very well done by most candidates.
(b) This question was reasonably well done by most candidates. Many candidates did not bother filling any circles and simply calculated the shortest route, they did not think that for a 4 mark question more was required. Many candidates filled only shortest path and not every circle. Some calculated the length of a Hamiltonian Path from A to I. Filling circle C correctly proved difficult for some. Some candidates mixed calculations of Hamiltonian Circuit distance and the Shortest Route on the same diagram.
(c) This question was reasonably well done by most candidates. Mostly well done. With more care many more candidates could have gained full marks – candidates are strongly recommended to double check their solution.
(d) This question was very well done by most candidates. Most candidates understood the differences between Hamiltonian and Eulerian paths and circuits.

**Question 18**
(a) This question was very well done by all the candidates who did it. Many candidates (around 1/3) missed out this question altogether. Candidates need to make sure that they answer every question on a section – where there is a mark make sure you have given an answer.
(b) This question was very well done by most candidates.
(c) This question was quite well done by most candidates. Some candidates did not follow the directions to do row reduction first and row and column reduction were swapped. Many candidates lost a half mark by not fully justifying their reason to not allocate after row reduction.
(d) This question was quite well done by most candidates. Some candidates incorrectly applied column reduction to columns by subtracting the next smallest number after 0. Final allocation was done well.

**Question 19**
(a) This question was quite poorly done by most candidates. Many candidates gave the EST of the next activity not the one asked for, even when they put the correct value on the diagram.
(b) This question was very well done by most candidates. Some candidates did not fill in all the required values on the diagram and simple adding mistakes were made.

This question was very well done by most candidates.
**Question 1** (Approximately 6 minutes)

A group of students and teachers was surveyed as to the type of smartphone they owned.

Of the 32 teachers interviewed, 19 owned an iPhone, whilst 40 students owned an iPhone and 50 students owned an Android phone.

(a) Use this information to complete the two-way table below.  

<table>
<thead>
<tr>
<th></th>
<th>Students</th>
<th>Teachers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPhone</td>
<td>40</td>
<td>19</td>
<td>59</td>
</tr>
<tr>
<td>Android phone</td>
<td>50</td>
<td>13</td>
<td>63</td>
</tr>
<tr>
<td>Total</td>
<td>90</td>
<td>32</td>
<td>122</td>
</tr>
</tbody>
</table>

(b) (i) Complete the table below to present this data in **percentage terms** and complete the segmented column graph below to display this information.  

<table>
<thead>
<tr>
<th></th>
<th>Students %</th>
<th>Teachers %</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPhone</td>
<td>44·4%</td>
<td>59·4%</td>
</tr>
<tr>
<td>Android phone</td>
<td>55·6%</td>
<td>40·6%</td>
</tr>
</tbody>
</table>

(ii) Briefly summarise your findings.  

Teachers prefer the iPhone.  

Students prefer the Android.  

More students own the Android (55·6%) compared to more teachers own the iPhone (59·4%).
**Question 2** (Approximately 17 minutes)

Leonie measured the depth of a car tyre tread (T), in mm, from when the tyre was new, until it had been driven on for a distance (D) of sixty thousand km. Her results are shown below.

<table>
<thead>
<tr>
<th>Distance driven (thousand km)</th>
<th>Tyre tread (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.0</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
</tr>
<tr>
<td>10</td>
<td>4.9</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
</tr>
<tr>
<td>30</td>
<td>3.2</td>
</tr>
<tr>
<td>40</td>
<td>2.3</td>
</tr>
<tr>
<td>50</td>
<td>2.0</td>
</tr>
<tr>
<td>60</td>
<td>1.8</td>
</tr>
</tbody>
</table>

(a) Which is the dependent (response) variable?  

Tyre tread

(b) State the linear equation for the line of best fit for this data. Give your answer to three decimal places.

\[ T = -0.073D + 5.655 \]  

2.D.P = \( \frac{1}{6} \) cm

(c) State the correlation coefficient for the linear equation in part (b). Explain what this means in terms of the modelled data.

\[ r = -0.9807 \]  

There is a strong negative correlation between the variables.

The further the distance driven, the smaller the tyre tread.

(d) Use algebra and your equation from part (b) to predict the distance driven when the tyre tread is 4.4 mm.

\[ 4.4 = -0.073D + 5.655 \]

\[ -1.255 = -0.073D \]

\[ D = \frac{-1.255}{-0.073} \]  

\[ = \frac{17.190}{1} \]

\[ = 17.19 \]  

**Question 2 continues.**
Question 2 (continued)

(e) Comment on the reliability, or otherwise, of your answer in part (d). (2 marks)

4.4 mm is within the data range and as a reasonable model would expect a reliable answer for such.

(f) Prepare a scaled residual plot for the linear model in part (b) on the grid below. (3 marks)

![Residual Plot]

(g) State whether or not the linear equation you have found in (b) is a good model for the data or not. Give reasons for your choice. (4 marks)

As there is clearly a definite pattern from the residual plot this indicates that another model might be a better choice.

There is also 3 points below where there is a 4 point or more deviation indicating that the best possible model.

Suggest not the best choice. Also see it as a linear graph it indicates that the data is not linear at all as has never been suggested an alternative is a better choice.
Question 3 (Approximately 13 minutes)

The quarterly water usage, Q1, Q2, Q3 and Q4 in kilolitres (kL), of a household over the years 2012 to 2014 is shown below. (Where Q1 is quarter 1, etc.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Quarterly average (kL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>195</td>
<td>95</td>
<td>53</td>
<td>65</td>
<td>102.0</td>
</tr>
<tr>
<td>2013</td>
<td>150</td>
<td>85</td>
<td>47</td>
<td>35</td>
<td>79.2</td>
</tr>
<tr>
<td>2014</td>
<td>140</td>
<td>99</td>
<td>37</td>
<td>25</td>
<td>75.2</td>
</tr>
</tbody>
</table>

(a) Include the Q1 to Q4 raw data for 2014 on the graph opposite. (1 mark)

(b) Use the quarterly average figures above to complete the Q4 column below and, hence, determine the seasonal index for Q4. Give your answers to three decimal places.

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Seasonal Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>1.912</td>
<td>0.931</td>
<td>0.520</td>
<td>0.637</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>1.893</td>
<td>1.073</td>
<td>0.593</td>
<td>0.442</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>1.860</td>
<td>1.316</td>
<td>0.492</td>
<td>0.332</td>
<td></td>
</tr>
<tr>
<td>Seasonal Index</td>
<td>1.888</td>
<td>1.107</td>
<td>0.535</td>
<td>0.471</td>
<td></td>
</tr>
</tbody>
</table>

(c) Deseasonalise the data for 2014 and include these points on the table below. Plot the deseasonalised data on the graph opposite. (2 marks)

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>103</td>
<td>86</td>
<td>99</td>
<td>138</td>
</tr>
<tr>
<td>2013</td>
<td>79</td>
<td>77</td>
<td>88</td>
<td>74</td>
</tr>
<tr>
<td>2014</td>
<td>74.2</td>
<td>89.4</td>
<td>69.2</td>
<td>53.1</td>
</tr>
</tbody>
</table>

Question 3 continues.
Question 3 (continued)

(d) Explain what has been the effect of deseasonalising the water usage data and describe the long term (secular) trend of the water usage over the three years. (3 marks)

Smooth out the water usage.
Water usage is showing a downward trend over the 3 years.

(e) From the information for the three years, 2012 to 2014, which quarter had the highest water usage on a deseasonalised basis? Explain what this means. (2 marks)

Q4 (2012) much higher water usage than expected for this quarter.

Question 3 continues.
Question 3 (continued)

The equation of the linear trend line of the deseasonalised data was found to be:

Water usage = $-3.822x + 110.750$, where $x$ is the quarter number

(f) Use this equation and another calculation to predict the actual water usage for this family in the first quarter of the year 2016. (3 marks)

\[
W = -3.822 \times 1 + 110.750 \\
= 45.776
\]

Actual water usage

\[
= 45.776 \times 1.888 \\
= 86.04 \text{ kL}
\]
Question 4 (Approximately 8 minutes)

Fatima starts a new job where her first year salary is $54,000. Each year after that her salary increases by $2,450.

(a) How much does Fatima earn in her second year in this job? (1 mark)

\[ 54,000 + 2,450 = 56,450 \]

(b) Write an arithmetic sequence rule for her salary. (2 marks)

\[ T_n = 54,000 + 2,450(n - 1) \]
\[ T_{n+1} = T_n + 2,450 \]
\[ = 51,550 + 2,450n \]
\[ + T_0 = 54,000 \]

(c) Algebraically, determine the year in which Fatima’s salary first reaches $71,150. (3 marks)

\[ 71,150 = 51,550 + 2,450n \]
\[ 19,600 = 2,450n \]
\[ n = \frac{19,600}{2,450} \]
\[ = 8 \]

She reaches $71,150 in the 8th year.

(d) Determine the total amount of money Fatima earns from when she starts her job to the end of the year she earns $71,150. (2 marks)

\[ S_n = \frac{n}{2} (2 \times 54,000 + (n-1) \times 2,450) \]
\[ = \frac{8}{2} (2 \times 54,000 + 7 \times 2,450) \]
\[ = 500,600 \]

\[ S_n = \frac{n}{2} (a + l) = 4 \times (54,000 + 71,150) \]
[\[ = 500,600 \]
Question 5 (Approximately 7 minutes)

The graph opposite shows how the number of bacteria in a culture increases over a period of five hours.

(a) What feature of this graph shows that the number of bacteria increases exponentially? (1 mark)

- The difference each hour is different
- The curve has a common ratio
- The curve increases sharply and clearly non-linear

(b) Using the two points indicated on the graph above, algebraically determine the rule for the geometric sequence representing the number of bacteria over this time period. (Give the common ratio to three decimal places.). (3 marks)

\[ T_1 = a \]
\[ T_3 = ar^2 \]
\[ = 1000 \]
\[ = 3000 \]
\[ a = 1000 \]
\[ r^2 = 3 \]
\[ r = \pm \sqrt{3} \]
\[ r = + 1.732 \text{ (but from graph)} \]
\[ T_n = 1000 (1.732)^{n-1} \]

Question 5 continues.
Question 5 (continued)

(c) At the end of which hour will the number of bacteria first exceed 100 000? Comment on the reliability of your prediction. (3 marks)

\[ 1000 \left(1.732\right)^{n-1} \geq 100000 \]

From calc \[ T_g \approx 80.98 \] \[ T_i \approx 140.25 \] during the 10th hour (after 10 hours)

As data only collected over 5 hour period the model is unknown after this period so unreliable as outside data

For use \[ 1000\left(1.732\right)^{n-1} = 100000 \text{ and solve} \]
Question 6 (Approximately 9 minutes)

A hardware shop sells 40 lawn mowers in one week and then 46 lawn mowers the following week.

(a) If an arithmetic sequence model applies to their sales, determine how many lawn mowers will be sold in the fifth week. (2 marks)

\[ T_n = 40 + 6(n - 1) \]

\[ = 34 + 6n \]  

\[ T_6 = 34 + 6 \times 5 = 64 \text{ lawn mowers} \]

(b) If a geometric sequence model applies to their sales, determine how many lawn mowers will be sold in the fifth week. (3 marks)

\[ T_n = a \cdot r^{n-1} \quad a = 40 \]

\[ r = \frac{46}{40} = 1.15 \]

\[ T_n = 40 \cdot (1.15)^{n-1} \]

\[ T_5 = 40 \cdot (1.15)^4 \]

\[ = 69.06 \]

\[ = 70 \text{ lawn mowers} \]

Question 6 continues.
Question 6 (continued)

(c) If the shop actually sold 364 lawn mowers in total from weeks 1 to 6, which of the sequence models in (a) and (b) seems to best model the lawn mower sales? Explain.

(4 marks)

- Arithmetic: Total sold from weeks 1 to 5
  
  i.e. 230 lawn mowers

  (1)

- Geometric sequence: Sold 250 in total

  (1)

- Given geometric sequence is 20 lawn mowers closer to total than AS
  
  i.e. GS seems to best model lawn mower sales

  (2)
Question 7 (Approximately 12 minutes)

A farmer estimates that he has 2 500 wallabies on his farm. The wallaby population increases by 20% each year, whilst the farmer removes 200 wallabies each year.

The first order difference (recurrence) equation that gives the number of wallabies on the farm at the end of each year (n = number of years) can be written as:

\[ W_{n+1} = 1.20 W_n - 200, \]  
where \( W_0 = 2 500 \) wallabies

(a) Use this equation to predict the number of wallabies at the end of the first and second years. 

End year 1: \[ 2500 \times 1.20 = 2800 \]

End year 2: \[ 2800 \times 1.20 = 3360 \]

(b) Prepare, on your calculator, a graph which shows the wallaby population for \( n = 0 \) to 5 years. Copy this graph onto the axes below.

Question 7 continues.
Question 7 (continued)

(c) If, instead, the farmer removes 800 wallabies each year, write the new **first order difference (recurrence) equation** below.

Graph these figures on the graph in part (b) for \( n = 0 \) to 5 years.

Describe how the wallaby numbers change now.  

\[
T_{n+1} = 1.20T_n - 800 \\
W_{n+1} = 1.20W_n - 800
\]

2 for graph

Wallaby numbers now decrease over time with a population of 267 at end of year 5

(d) **Algebraically**, determine how many wallabies the farmer would have to remove each year if the population was to remain at (a constant) 2,500 at the end of every year.  

\[
1.20 \times 2500 = 3000 \\
\therefore d = 500 \\
\therefore \text{Remove} \ 500 \ \text{each year}
\]

\[
W_{n+1} = 1.20W_n - 500
\]

0 or 20% of 2500 = 500  
\[\therefore \text{if increase is 500 need to remove 500}\]
At the beginning of March, Sophie had $3000 in her bank account.

On 25 March Sophie withdrew $1500 and on 2 May she deposited $2000.

(a) If the account pays interest of 3.65% p.a. on a daily balance, how much interest will she have earned at the end of 30 June? (4 marks)

\[ \text{March 1st} \quad \text{Dep with} \quad \text{Balance} \quad \text{days} \quad \text{Interest} \]
\[ 25th \quad 11 \quad 1500 \quad 1500 \quad 52 \quad \text{days=121} \]
\[ \text{May 2nd} \quad 2000 \quad 3500 \quad 50 \quad \text{=33.55} \]
\[ \text{Total} = 33.55 \]

(b) What will be the balance of Sophie’s account at the end of 30 June? (1 mark)

\[ \text{Balance} = \ 2500 + 33.55 \]
\[ = \ 2533.55 \]
Question 9 (Approximately 8 minutes)

Aisha wants to have $17,500 in a bank account after 2 years so that she can buy a new car.

(a) Algebraically, calculate the amount of money she needs to invest to have $17,500 if interest is 4.40% p.a., compounded monthly. (3 marks)

\[
A = P \left(1 + \frac{0.044}{12}\right)^{12n}
\]

\[
A = 17,500
\]

\[
P = \frac{17,500}{1.0036666666666667}
\]

\[
i = \frac{0.044}{12}
\]

\[
= \frac{116,028.29}{2} = 58,014.15
\]

(b) Aisha finds out that her $17,500 new car depreciates at rate of 20% p.a. on a reducing balance basis. Determine what her car will be worth after 5 years of ownership. (2 marks)

\[
A = P \left(1 - \frac{0.20}{12}\right)^{12n}
\]

\[
= 17,500 \left(1 - 0.2\right)^5
\]

\[
= 5,734.40
\]

(c) Aisha sells her car after 5 years for the figure in part (b) and puts this money towards another new car.

If the price of a comparable new car increased at a rate of 1.3% p.a. over the same time period, determine how much extra Aisha would need in order to buy such a car in 5 years time. (3 marks)

\[
A = P \left(1 + \frac{0.013}{12}\right)^{12n}
\]

\[
= 17,500 \left(1 + 0.013\right)^5
\]

\[
= 18,667.47
\]

Extra = 18,667.47 - 5,734.40

= 12,933.07
**Question 10** (Approximately 10 minutes)

A company deposits $2,000 each quarter into an annuities account which earns 4.80% p.a., compounded monthly.

(a) Determine the balance of this annuities account after 5 years. (4 marks)

\[
\begin{align*}
\text{Begin} & \quad N = 4 \times 5 = 20 \\
\text{I\%} & = 4.8\% \\
\text{PV} & = 0 \\
\text{PMT} & = -2000 \\
FV & = ? \\
\text{P\%\%} & = 4 \\
\text{c\%\%} & = 12
\end{align*}
\]

(b) After five years, the company invests $45,000 of the money earned from part (a) into another account that pays an interest rate of 5.0% interest compounding annually.

If the interest earned each year is awarded as an annual scholarship, determine the value of the annual scholarship. (2 marks)

\[
\begin{align*}
\hat{I} & = P \times i \\
& = 45000 \times 0.05 \\
& = 2250
\end{align*}
\]

Question 10 continues.
Question 10 (continued)

(c) If the annual scholarship was instead $5000 each year, determine how long this could be awarded for.

\[ N = 7 \quad \Rightarrow \quad \text{awarded for} \]
\[ I\% = 6.0\% \quad \Rightarrow \quad 12 \text{ years} \]
\[ PV = 45000 \]
\[ PMT = -5000 \]
\[ FV = 0 \]
\[ \frac{PV}{y} = 1 \]
\[ \frac{PV}{y} = 1 \]
Question 11 (Approximately 13 minutes)

Bill owes $11 000 on his credit card and he is charged an interest rate of 19.00% p.a., compounded monthly, on this debt.

If Bill was to repay this credit card debt at the repayment figure of $183.14 per month, he would pay the debt after 16 years exactly.

(a) Determine how much interest Bill will pay on his credit card debt of $11 000 over the 16 years. (2 marks)

\[
\text{Total Paid} = 183.14 \times 16 \times 12
\]

\[
= 35162.88
\]

\[
\text{Int} = $24162.88
\]

(b) Using an appropriate formula, show that Bill owes $10 741 (to the nearest dollar) after 2 years of repayments. (3 marks)

\[
P = \frac{R \left[1 - (1 + \frac{a}{12})^{-n}\right]}{\frac{a}{12}}
\]

\[
R = 183.14
\]

\[
= 183.14 \left[1 - (1 + \frac{0.19}{12})^{-16}\right]
\]

\[
= 183.14 \left[1 - (1 + \frac{0.19}{12})^{-12}\right]
\]

\[
= 183.14 \left[1 - (1 + 0.0158)^{-12}\right]
\]

\[
= 183.14 \left[1 - (1 + 0.0158)^{-12}\right]
\]

\[
= \frac{183.14 \times 0.063}{0.063}
\]

\[
= $10,741 \text{ to nearest } $1
\]

(c) After two years, Bill is given the option of paying off this $10 741 at an interest rate of 1.99% p.a., compounded monthly. He first has to pay a transfer fee of 1.25% on the figure from part (b).

Including the transfer fee of 1.25%, if Bill takes up this option, how much will he owe now? (2 marks)

\[
\text{Transfer fee} = 1.25\% \times 10,741 = 134.025
\]

\[
\text{Owe} \quad $10,741.25
\]

Question 11 continues.
Question 11 (continued)

(d) Bill decides to repay $500 every month towards paying off the amount owing in part (c) at the 1.99% p.a. compounded monthly interest rate.

Determine how much time and how much money Bill will save by repaying the debt this way in comparison with the original payment terms given at the beginning of the question. (6 marks)

\[ N = 7 \]
\[ N = 22.017 \text{ months} \]
\[ I\% = 1.99 \]
\[ PV = 10527.526 \]
\[ PMT = -500 \]
\[ FV = 0 \]
\[ \frac{y}{y} = 12 \]
\[ \frac{1}{y} = 12 \]

16.8 months = 22.017 ≈ 14.583 months ≈ 146 months.

Time saved: \[ 14 \text{ years } 11 \text{ months} \]
\[ = 12 \text{ years } 1 \text{ month} \]
Money saved: \[ \text{Total saved} = 2 \times 12 \times 1.122014 \]
\[ = 22.017 \times 500 \]
\[ = 4395.36 + 11085 \]
\[ = \$15480.36 \]
\[ \text{Saved} = 28162.88 - 15480.36 = \$12682.52 \]
Question 12 (Approximately 6 minutes)

A cargo ship sails due east from Christchurch (44.0°S, 172.7°E) to the Chatham Islands (44.0°S, 176.0°E).

(a) What distance, in nautical miles, does the cargo ship sail? (2 marks)

\[ D = 60 \times \cos \theta \]
\[ = 60 \times 3.3 \times \cos 44 \]
\[ = 14.2 \text{ n.m.} \]

(b) If the cargo ship left Christchurch at 7:10 pm on 2 May and travelled at an average speed of 16.0 knots, determine the time it arrived in the Chatham Islands. (Note that both Christchurch and the Chatham Islands are in the same time zone). (4 marks)

\[ \text{Travel time} = \frac{142.4}{16} = 8.09 \text{ hours} \]
\[ = 8 \text{ hours 54 mins} \]

\[ \text{Arrival time} = 7:10 + 8 \text{ hours 54 mins} = 4:04 \text{ am, next day} \]

2 marks for travel time in hours mins
2 marks for correct arrival time calculation
Question 13 (Approximately 5 minutes)

A dark shade cloth ‘sail’ has the dimensions shown below.

(a) Determine the area of the smaller triangle above with side lengths of 5.0, 3.0 and 7.1 m. Give your answer to two decimal places.

\[ A = \frac{1}{2} \times 5.0 \times 3.0 \times \sin(7.1) \]

\[ A = \frac{1}{2} \times 5.0 \times 3.0 \times 0.94 \]

\[ A = 7.029 \text{ m}^2 \]

(b) Determine the area of the dark shade cloth ‘sail’. Give your answer to two decimal places.

Area of large \( \triangle = \frac{1}{2} \times 6 \times 7.1 \times 6 \)

\[ = 21.03 \text{ m}^2 \]

Area of shade cloth = 21.03 - 6.28

\[ = 14.75 \text{ m}^2 \]
Question 14 (Approximately 14 minutes)

Gary and Kathryn flew from Buenos Aires (35°S, 58°W) to Melbourne (38°S, 145°E) via the shortest possible route.

They departed Buenos Aires at 11:00 am on Wednesday, 25 July. The flight path their plane took is shown on the map below.

(a) Determine the standard time difference between Melbourne and Buenos Aires. (2 marks)

\[
\begin{align*}
M & = +10 \text{ UTC} \\
BA & = \frac{58}{15} = 3.86 \therefore -4 \text{ UTC}
\end{align*}
\]

(b) If Kathryn made a phone call from Buenos Aires at 9:45 am on Wednesday, 25 July to a friend in Melbourne, at what Melbourne (EST) time did her friend receive the phone call? (2 marks)

\[
\begin{align*}
B.A & \quad M \\
9:45 \text{ am} +11 & \quad 11:45 \text{ pm} \\
& 14 \text{ hours ahead}
\end{align*}
\]

Question 14 continues.
Question 14 (continued)

(c) Briefly explain why the flight path shown in the diagram opposite appears to be very curved. The flight path follows the great circle route. The Earth is spherical and when projected onto a flat surface at either pole, the distance is exaggerated, giving a much more curved appearance than it actually is. (2 marks)

(d) Determine the distance (in km) that the plane flew from Buenos Aires to Melbourne. (4 marks)

\[ D = 2 \pi R \theta \]
\[ D = 2 \pi R \times 0.350 \]
\[ \cos \theta = \sin(-30) \sin(-30) + \cos(-30) \cos(-30) \cos(157) \]
\[ \cos \theta = 1.0 \times 0.8944 \]
\[ \theta = 103.9^\circ \]
\[ D = 2.54 \times 0.350 \]
\[ D = 11559 \text{ km} \]

(e) If their plane arrived in Melbourne at 3:27 pm on Thursday, 26 July, determine the average speed of the plane, in km per hour. (4 marks)

\[ S = \frac{D}{T} \]
\[ T = 15:27 - 1 = 14 \text{ hours 27 minutes} \]
\[ T = \frac{11000 + 14}{1500} \]
\[ S = \frac{11559}{14.45} \]
\[ S = 799.9 \text{ km/h} \]
\[ S = 800 \text{ km/h} \]
**Question 15** (Approximately 11 minutes)

Andrew (A), Brenda (B) and Chris (C) are standing on a level field. Andrew (A) notices that Brenda (B) is 77 metres away from him at a bearing of 064°T, whilst Chris (C) is 42 m away from him at a bearing of 140°T as shown in the diagram below.

![Diagram showing the positions of Andrew, Brenda, and Chris](image)

(a) Complete the diagram above with the information given in the question and hence show that the angle at A (angle CAB) is 76°. (3 marks)

\[ \theta = 140° - 64° = 76° \]

(b) Determine the distance from Brenda to Chris. Give your answer to one decimal place. (2 marks)

\[ d^2 = 7.7^2 + 4.2^2 - 2 \times 7.7 \times 4.2 \cos 76° \]

\[ = 6.12 \text{ m} \]

\[ d = 7.8 \text{ m} \]

**Question 15 continues.**
Question 15 (continued)

(c) Determine the angle at B (angle ABC) and hence determine the **true bearing** from Brenda to Chris. Give your answers to the **nearest minute**. (6 marks)

\[
\frac{\sin \theta}{42} = \frac{\sin 76}{78.3}
\]

\[
\sin \theta = \frac{42 \times \sin 76}{78.3}
\]

\[
\theta = 31.036^\circ
\]

\[
\theta = 31^\circ 22' 1''
\]

**Angle at B:** 31° 22' 1''

**True bearing from Brenda to Chris:**

\[
\theta = 64^\circ - 31^\circ 22' 1'' \leq (1)
\]

\[
= 32^\circ 38' 57'' \leq (1)
\]

\[
= 32^\circ 38' 1'' \leq (1)
\]
Question 16 (Approximately 7 minutes)

The network below represents the water flow, in litres per minute, in a series of pipes.

(a) Identify the source and the sink of this network. (1 mark)

Source: .................................................................

Sink: .................................................................

(b) Give the values of the cuts I, II, III, IV and V in the spaces provided in the above network. Hence, determine the maximum flow of the network. (3 marks)

Maximum flow: ........................................ 680 l/min

(c) In order to increase the water flow in this system, it is planned to upgrade the pipe section CF. What is the maximum worthwhile upgrade to this pipe section and what would the new maximum flow rate then be? Explain. (3 marks)

Upgrade to 400 l/min

As ABC only has 380 l/min

This is max amount in ABC

Max available is 400

But as max is 780 l/min, this is above

What is required so increase by 100 l/min
to 380 l/min
Question 17 (Approximately 11 minutes)

The network below shows the roads, in km, that exist between nine farms, A to I.

(a) Describe a Hamiltonian Circuit that starts and finishes at farm A. What distance does this circuit cover? (2 marks)

```
A -> B -> C -> F -> I -> H -> G -> D -> E -> B -> A
```

Distance covered: $15 + 24 + 17 + 15 + 12 + 11 + 12 + 26 = 138$ km

(b) A delivery truck travels from farm A to farm I. Using an algorithmic approach on the network diagram above, determine the shortest route the truck can take and the distance that it travels. (4 marks)

Shortest route from A to I: $A -> B -> C -> F -> I$ (1)

Distance covered: $15 + 24 + 17 + 15 + 12 + 11 + 12 + 26 = 138$ km (1)
Question 17 (continued)

(c) An electricity company needs to upgrade its connections between these farms. Sketch on the diagram below, the **minimum spanning tree** and hence determine the **minimum distance** covered.

Minimum distance covered: \[6 + 8 + 17 + 10 + 10 + 6 + 11 + 10\] \[= 78 \text{ km}\]

(d) Hire cars can only travel on sealed roads in Tasmania. The reduced network diagram below shows only sealed roads between the farms.

State an **Eulerian path** where a hire car can travel along each and every road exactly once.

\[B - A - C - B - E - G - F - D - C - E - D\] or \[D - E - G - F - D - C - A - B - E - C - B\]

or \[D\]
Question 18 (Approximately 9 minutes)

The bipartite graph below, shows the favourite subjects of four students, Anna, Bertrand, Cam and David.

(a) David lists his favourite subjects as General Maths and English Communications. Include this information on the bipartite graph. (1 mark)

(b) Which is a favourite subject of both Bertrand and Cam? (1 mark)

As part of a General Maths activity, Anna, Bertrand, Cam and David participate in a competition that has four different activities, E, F, G and H.

The time, in minutes, that each student took to complete each activity is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>12</td>
<td>13</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Bertrand</td>
<td>10</td>
<td>8</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Cam</td>
<td>7</td>
<td>9</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>David</td>
<td>8</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

Question 18 continues.
**Question 18 (continued)**

In order to best assign these students to an activity it is necessary to perform a row reduction then a column reduction.

(c) Complete the **row reduction** process in the table below. (3 marks)

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Bertrand</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Cam</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>David</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Explain why it is **not possible** to complete the allocation of activities after this step.

- Need 4 lines or (4x4) matrix.

(d) Complete the **column reduction** below and then allocate each student to a task. (4 marks)

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>O</td>
</tr>
<tr>
<td>Bertrand</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Cam</td>
<td>1</td>
<td>3</td>
<td>O</td>
<td>3</td>
</tr>
<tr>
<td>David</td>
<td>O</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Allocation:  
- Anna - H  
- David - E  
- Cam - G  
- Bertrand - F

Total time taken:  
\[8 + 8 + 6 + 10 = 32\text{ mins}\]
Question 19 (Approximately 9 minutes)

The times taken, in days, for the tasks A to L to be completed are shown in the project network below. Note that some earliest start times (EST) are shown.

(a) Determine the **earliest starting times** (EST) for activities F and G. (2 marks)

\[
\begin{align*}
F & : 24 \text{ days} \\
G & : 22 \text{ days}
\end{align*}
\]

(b) Complete all of the (time keeping) boxes in the project network above.

Use this information to determine the critical path and indicate the minimum time to complete this project. (5 marks)

Critical path: \[A, C, G, H, \ldots, L\] (1)

Minimum time to complete the project: 57 days (1)

Question 19 continues.
Question 19 (continued)

(c) Determine by how long the following two activities could be delayed before the minimum time to complete this network was affected. 

(2 marks)

Activity H: ........................................... 0 time part of critical path .........................................................

Activity K: ........................................... 11 days ..........................................................

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