MATHEMATICS – METHODS

Senior Secondary

Subject Code: MTM315114

External Assessment

2015

PART 1
Calculators are NOT allowed to be used

Time: 80 minutes

On the basis of your performance in this examination, the examiners will provide a result on the following criteria taken from the syllabus statement:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
<th>For Marker Use Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Demonstrate an understanding of polynomial, hyperbolic, exponential and logarithmic functions.</td>
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<td>8</td>
<td>Demonstrate an understanding of binomial, hypergeometric and normal probability distributions.</td>
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</tr>
</tbody>
</table>
CANDIDATE INSTRUCTIONS

You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.

This part of the examination is worth 60 marks in total. Each section is worth 12 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

You may use the 2015 External Examination Information Sheet for Mathematics Methods provided with the examination throughout the examination. No other written material is allowed into the examination.

For questions worth 1 or 2 marks, you do not have to show any working.

For questions worth 3 or more marks, you must show relevant working.

You must write your answers and any working in the spaces provided on the examination paper.

All written responses must be in English.
Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

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**Question 1**

Consider the polynomial function \( f(x) = 7 - 2x^3 \).

(a) Determine the inverse function \( f^{-1}(x) \). (3 marks)

(b) Explain why \( f(x) \) has an inverse function. (1 mark)

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**Question 2**

Determine the coefficient of the \( x^2 \) term in the expansion of \( (2x - 1)^5 \). (3 marks)

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**Section A continues.**
Section A (continued)

Question 3

Given that \( f(x) = x^2 + 1 \) and \( g(x) = \frac{4x}{x+1} \).

(a) Write an expression for \( g(f(x)) \). (2 marks)

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(b) Hence or otherwise determine \( g(f(-1)) \). (1 mark)

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Section A (continued)

Question 4

The graph of \( f(x) \) is shown below.

From the four options below indicate the graph that represents \( g(x) = -2f(x+1) \).

For Marker
Use Only
Answer ALL questions in this section.

This section assesses Criterion 5.

Question 5 (3 marks)

(a) Convert 165° to radians. Write your answer in simplest form.

(b) Convert \( \frac{3\pi}{8} \) to degrees. Write your answer in simplest form.

Question 6 (3 marks)

If \( \theta = \frac{\pi}{6} \) then determine:

(a) \( \sin \theta \)

(b) \( \cos(\pi - \theta) \)

(c) \( \tan\left(\frac{\pi}{3} + \theta\right) \)

Section B continues.
Section B (continued)

Question 7  
(2 marks)

Determine the number of solutions to \( \cos\left(\frac{\pi \theta}{4}\right) = -0.3 \) on the domain \( \theta \in [-4, 10] \).


Question 8  
(2 marks)

In an exam a student was asked to sketch the graph of \( y = 4 \sin\left(\frac{\pi}{6}(x - 5)\right) + 2 \) on the domain \( x \in [-5, 15] \). The student’s answer is shown below.

Identify which of the following features are correct or incorrect in the graph.

Amplitude: CORRECT INCORRECT
Period: CORRECT INCORRECT
Vertical Translation: CORRECT INCORRECT
Phase Shift: CORRECT INCORRECT

Question 9  
(2 marks)

Given that \( \sin \theta = 0.2 \) then use the double angle formula to determine \( \cos 2\theta \).
Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

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**Question 10**

(4 marks)

Find the derivative with respect to \( x \) of:

(a) \( y = e^{2x} + x^2 \log_e 4 \).

(b) \( f(x) = 3 \sqrt{x} + \frac{1}{x} \).
Section C (continued)

Question 11  
(2 marks)

Consider the graph of \( f(x) \) below. State for what value(s) of \( x \) the derivative is undefined.

\[ ... \]

Question 12  
(3 marks)

Given that \( f(x) = \frac{\sin 2x}{\cos x} \) then determine \( f''\left(\frac{\pi}{3}\right) \).

\[ ... \]

\[ ... \]

\[ ... \]

\[ ... \]

Section C continues.
Section C (continued)

Question 13

\( f(x) \) has a domain of \((0,4) \setminus \{2\}\).

The derivative graph, \( f'(x) \) with domain \((0,4) \setminus \{2\}\) is shown.

(a) \( f(x) \) has a stationary point at \( x = 1 \). State its nature.

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(b) Describe the shape of \( f(x) \) as \( x \) approaches 4.

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(c) Identify which of the following graphs is \( f(x) \).

A

B

C

D
Section D

Answer ALL questions in this section.

This section assesses Criterion 7.

Question 14 (2 marks)

Determine the integral of \( e^x + \log_e 2 + \frac{3}{x^4} \).

Question 15 (4 marks)

(a) Determine \( \int \left( 1 + \frac{1}{x} \right)^3 \, dx \).

(b) Hence determine \( \int_1^e \left( 1 + \frac{1}{x} \right)^3 \, dx \).

Section D continues.
Section D (continued)

Question 16

If \( f'(x) = (2x - 1)^5 \) and \( f(0) = 1 \) then determine \( f(x) \).

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Question 17

Given that \( \int_{1}^{4} (3f(x) + 2x) \, dx = 7 \) then determine \( \int_{1}^{4} f(x) \, dx \).

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Section E

Answer ALL questions in this section.

This section assesses Criterion 8.

Question 18  
(2 marks)

Consider the following scenario.

A little boy reached into a bag of jellybeans and pulled out five, none of which was blue. His mother said that the chance of getting all blue jellybeans was only 1%.

(a) Define the random variable described in the scenario.

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(b) Write a probability statement, based on the scenario, in terms of your random variable.

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Section E continues.
Section E (continued)

Question 19  
(4 marks)

A pair of identical six-sided dice have a star on four sides and two sides are blank. A player rolls both dice at the same time.

If both dice show a star the player gets three points. If only one of the dice shows a star the player gets one point. If both dice are blank the player gets zero points.

Let $X$ be the number of points awarded when the pair of dice are rolled.

(a) Complete the probability distribution table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr $(X = x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Hence determine the expected number of points per roll of the pair of dice.

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Question 20  
(3 marks)

The time taken by students to complete a set of problems is normally distributed with a mean of 15 minutes and a standard deviation of 2 minutes and 30 seconds. A group of 40 students undertook the set of problems.

How many students took between 10 minutes and 15 minutes?

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Section E continues.
Section E (continued)

Question 21  

In an experiment, a biased coin (a coin without equal probability of getting heads and tails) was flipped 20 times. The number of times a head appears was recorded. This experiment was repeated many times to obtain the probability distribution shown below, where $\mu = \frac{90}{7}$ and $\sigma = \frac{15}{7}$.

![Probability distribution graph](image)

In a new set of experiments the same coin was flipped 80 times per experiment.

(a) By considering what happens to the mean and standard deviation when $n$ is increased, determine the mean and standard deviation of the new set of experiments.

(b) Identify which of the following probability distributions corresponds to this new set of experiments?

A  

B  

C  

D
Tasmanian Certificate of Education

MATHEMATICS – METHODS

Senior Secondary

Subject Code: MTM315114

External Assessment

2015

PART 2
Calculators are allowed to be used

Time: 100 minutes

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<tr>
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Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

**Question 22**

The graph of \( f : [2, 5) \setminus \{3\} \to \mathbb{R}, \quad f(x) = \frac{2}{(x - 3)^2} + 1 \) has a restricted domain of \([2, 5) \setminus \{3\}\).

(a) Find the intercepts of \( f \), if any.

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(b) Calculate the range of \( f \).

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**Question 23**

Sketch a possible graph of \( y = (x - a)(x + b)^2 \) where \( a \) and \( b \) are both positive. Label the zeros and the y-intercept (in terms of \( a \) and \( b \)).

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Section A continues.
Section A (continued)

Question 24  
(4 marks)

Determine algebraically the centre and radius of the circle defined by:

\[ x^2 + y^2 - 4x + 6y - 3 = 0 \]

Question 25

(a) Use the change of base formula to show that \(4 \log_4 A - 3 \log_8 A = \log_2 A\).  
(2 marks)

(b) Hence solve algebraically \(4 \log_4 (1 - x) - 3 \log_8 (1 - x) + \log_2 (5 - x) = 5\)  
(4 marks)
Section B

Answer ALL questions in this section.

This section assesses **Criterion 5**.

**Question 26**

(4 marks)

The angle $\theta$ is shown on the unit circle below. On the same diagram draw and label:

(a) the angle $\frac{3\pi}{2} + \theta$,
(b) the length $\sin \theta$,
(c) the length $\cos(\pi - \theta)$,
(d) the length $\tan \theta$.

Spare diagram

Section B continues.
Section B (continued)

Question 27

In this question \( \sin \theta = \frac{15}{17} \) and \( 0 < \theta < \frac{\pi}{2} \).

(a) Use the fact that \( \sin^2 \theta + \cos^2 \theta = 1 \) to find the exact value of \( \cos \theta \). (3 marks)

(b) Hence, use the addition theorem to determine the exact value of \( \sin \left( \theta + \frac{\pi}{6} \right) \). (3 marks)

Section B continues.
Section B (continued)

Question 28 (6 marks)

A boat left Hobart at high tide at 2:00 pm on Tuesday. The high tide at this time was measured at 1.7 m.

The boat returned to Hobart at low tide at 11:00 am on Wednesday. The low tide at this time am was measured at 0.5 m.

While the boat was out to sea there was one low tide and one high tide.

*Note*: Low tide is the time when the local water level of the ocean is at its lowest. High tide is the time when the local water level of the ocean is at its highest.

(a) Determine an equation that models the tidal height as a function of time (in hours) since midday (12 noon) on Tuesday.

(b) Use your model to determine at what time(s) of day on **Wednesday** the tidal height is exactly 0.8 m. Give the time(s) in hours and minutes.
Section C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

**Question 29**

(a) Determine algebraically the coordinates of the **three** points of \( f(x) = x^4 - 4x^3 + 4x^2 - x \) which have a gradient of \(-1\).

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(b) By considering the tangents through these points, show that **two** of the points lie on the same tangent.

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**Section C continues.**
Section C (continued)

Question 30

(a) Find the derivative of \( y = x^4 e^{4x} \).

(b) Hence find the coordinates of, and classify the nature of, all stationary points on the graph of \( y = x^4 e^{4x} \).

Section C continues.
Section C (continued)

Question 31

A secant is a straight line which passes through two points on a curve. The points are labelled \((x_1, y_1)\) and \((x_2, y_2)\) in the adjacent diagram and the secant is the straight dashed line.

(a) For the curve \(f(x) = x^3\) determine the \(y\) coordinate of each pair of points given in the table below and hence the slope of the secant through those two points. The first two have been completed as an example. (3 marks)

<table>
<thead>
<tr>
<th>(x_1)</th>
<th>(y_1)</th>
<th>(x_2)</th>
<th>(y_2)</th>
<th>slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>5</td>
<td>125</td>
<td>21</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
<td>64</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{3}{2})</td>
<td>(\frac{5}{2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2-h)</td>
<td>(2+h)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Determine the limit as \(h \to 0\) of the slope between the points \(x_1 = 2-h\) and \(x_2 = 2+h\). Show that this is equal to the gradient of the tangent to the curve at the point \(x = 2\). (3 marks)
Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

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**Question 32**

(a) Determine algebraically the x-values of the points of intersection of:

\[ f(x) = x^3 - 4x^2 + x + 2 \]
\[ g(x) = 2x^2 - 7x + 2 \]

(b) Hence find the area enclosed between the curves \( f(x) \) and \( g(x) \).
Section D (continued)

Question 33

Find the derivative of \((\log_e x)^2\) and hence determine \(\int_1^e \frac{\log_e x}{x} \, dx\)

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Question 34

\(g(x)\) is a function which is both integrable and differentiable. Two students carry out the following:

• Student A: Integrated \(g(x)\) then differentiated the result.
• Student B: Differentiated \(g(x)\) then integrated the result.

(a) Do they get the same expression?

YES    NO

(b) If they are equal justify your answer. If they are not equal explain how they differ.

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Section D continues.
Section D (continued)

Question 35  

The length, \( L \), of a curve \( f(x) \) between the points \( x = a \) and \( x = b \) is given by:

\[
L = \int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx
\]

Determine the length of the curve \( f(x) = x^{\frac{3}{2}} \) from \( x = 0 \) to \( x = 13 \).
Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

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**Question 36**  
(2 marks)

In a test a ‘B’ rating was awarded to all students who earned between 60 and 75 marks. The marks were normally distributed with an average of 50 marks and a standard deviation of 15 marks.

What percentage of students received a ‘B’ rating?

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**Question 37**  
(3 marks)

In a race the fastest 10% of runners received a gold medal. The next fastest 10% of runners after that received a silver medal. Bronze medals were given to the next fastest 10% of runners after that.

The finishing times were normally distributed. The average time to complete the race was 45 minutes and the standard deviation was 8 minutes.

Between what two times did a runner finish if they received a bronze medal?

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**Section E continues.**
Section E (continued)

Question 38

Two students wanted to explore the difference between different types of probability distributions. They filled a bag with 30 green marbles and 70 white marbles. They each carried out a different set of experiments using the bag of marbles.

Student A took out 20 marbles at once and recorded the number of green marbles.

Student B took out one marble and wrote down the colour then placed the marble back in the bag. Student B repeated this for a total of 20 times then counted up the number of times green was recorded.

(a) Complete the table below. The mean and variance for Student A has been provided. 

(2 marks)

<table>
<thead>
<tr>
<th>Student</th>
<th>Type of Distribution</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>6</td>
<td>\frac{112}{33}</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Determine the probability of recording six green marbles for: 

(2 marks)

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Student A ............................................................................................................................................

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Student B ............................................................................................................................................

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(c) If the bag contained 10 000 marbles (3 000 green and 7 000 white), then the answers for both students would have been almost the same.

Explain why this would happen. 

(2 marks)

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Section E continues.
Section E (continued)

Question 39

A box contains $r$ red marbles and $b$ blue marbles.

A machine takes three marbles from the box at the same time and records the number of red marbles before returning all three marbles to the box. A random variable, $X$, is the number of red marbles observed.

(a) Draw a line to connect each probability statement on the left to the corresponding algebraic expression on the right. (2 marks)

- \[ \Pr(X = 0) \quad \frac{r \binom{0}{b} \binom{3}{b}}{r + b \binom{3}{b}} = \frac{b(b-1)(b-2)}{(b+r)(b+r-1)(b+r-2)} \]
- \[ \Pr(X = 1) \quad \frac{r \binom{1}{b} \binom{2}{b}}{r + b \binom{3}{b}} = \frac{r(r-1)(r-2)}{(b+r)(b+r-1)(b+r-2)} \]
- \[ \Pr(X = 2) \quad \frac{r \binom{2}{b} \binom{1}{b}}{r + b \binom{3}{b}} = \frac{3br(b-1)}{(b+r)(b+r-1)(b+r-2)} \]
- \[ \Pr(X = 3) \quad \frac{r \binom{3}{b} \binom{0}{b}}{r + b \binom{3}{b}} = \frac{3br(r-1)}{(b+r)(b+r-1)(b+r-2)} \]

(b) The box is filled with twice as many blue marbles as red marbles. This means $b = 2r$.

The machine repeatedly draws three marbles and records the number of red marbles. Over a very large number of draws it observes the following probabilities.

Determine the number of red and blue marbles. (3 marks)

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr(X = x)$</td>
<td>14/55</td>
<td>28/55</td>
<td>12/55</td>
<td>1/55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<td>28/55</td>
<td>12/55</td>
<td>1/55</td>
</tr>
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FUNCTION STUDY

Quadratic Formula: If \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Graph Shapes:

- **Quadratic**: \( y = a(x-h)^2 + k \)
- **Cubic**: \( y = a(x-h)^3 + k \)
- **Hyperbola**: \( y = \frac{a}{x-h} + k \)
- **Truncus**: \( y = \frac{a}{(x-h)^2} + k \)

Graphical Transformations:

The graph of:
- \( y = -f(x) \) is a reflection of the graph of \( y = f(x) \) in the x axis
- \( y = f(-x) \) is a reflection of the graph of \( y = f(x) \) in the y axis
- \( y = af(x) \) is a dilation of the graph of \( y = f(x) \) by factor \( a \) in the direction of the y axis
- \( y = f(ax) \) is a dilation of the graph of \( y = f(x) \) by factor \( \frac{1}{a} \) in the direction of the x axis
- \( y = f(x+b) \) is a translation of the graph of \( y = f(x) \) by \( b \) units to the left
- \( y = f(x) + b \) is a translation of the graph of \( y = f(x) \) by \( b \) units upwards

**Index Laws**

- \( a^x \times a^y = a^{x+y} \)
- \( a^x + a^y = a^{x+y} \)
- \( (a^x)^y = a^{xy} \)
- \( (a^x)^{\frac{1}{y}} = a^{x/y} \)
- \( (a^x)^y = (a^y)^x \)

**Log Laws**

- \( \log_a xy = \log_a x + \log_a y \)
- \( \log_a \left( \frac{x}{y} \right) = \log_a x - \log_a y \)
- \( \log_a x^n = n \log_a x \)
- \( \log_a x = \frac{\log_b x}{\log_b a} \)

**Useful log results**

Definition: If \( y = a^x \) then

- \( \log_a y = x \)
- \( \log_a 1 = 0 \)
- \( \ln 1 = 0 \)
- \( \log_a a = 1 \)
- \( \ln e = 1 \)

**Inverse Functions**

- \( f^{-1} \left( f(x) \right) = x \)

**Binomial Expansion**

- \( (x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \ldots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n \)
CIRCULAR FUNCTIONS

Conversion:
To convert from radians to degrees multiply by \( \frac{180}{\pi} \)
To convert from degrees to radians multiply by \( \frac{\pi}{180} \)

Basic Identities:
\[
\begin{align*}
\sin^2 x + \cos^2 x &= 1 \\
\tan x &= \frac{\sin x}{\cos x} \\
\cot x &= \frac{1}{\tan x} \\
\sec x &= \frac{1}{\cos x} \\
\csc x &= \frac{1}{\sin x}
\end{align*}
\]

Multiple Angle Formulae:
\[
\begin{align*}
\sin(A + B) &= \sin A \cos B + \cos A \sin B \\
\cos(A + B) &= \cos A \cos B - \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\
\tan (A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
\tan (A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\
\tan 2A &= \frac{2\tan A}{1 - \tan^2 A}
\end{align*}
\]

Exact Values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \frac{\pi}{6} )</th>
<th>( \frac{\pi}{4} )</th>
<th>( \frac{\pi}{3} )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \frac{3\pi}{2} )</th>
<th>2( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x )</td>
<td>0</td>
<td>1</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( \cos x )</td>
<td>1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( \tan x )</td>
<td>0</td>
<td>( \frac{\sqrt{3}}{3} )</td>
<td>1</td>
<td>( \sqrt{3} )</td>
<td>undefined</td>
<td>0</td>
<td>undefined</td>
</tr>
</tbody>
</table>

Trigonometric Graphs:

Graphical Transformation:
The graph of \( y = a \sin(n(x+b)) + c \) or \( y = a \cos(n(x+b)) + c \) has:
- amplitude: \( |a| \)
- period: \( \frac{2\pi}{n} \)
- phase shift: \( b \) (shift of \( b \) units to the left)
- vertical shift: \( c \) units upwards

The graph of \( y = a \tan(n(x+b)) + c \) has:
- dilation: by factor \( a \) in the direction of the \( y \) axis
- period: \( \frac{\pi}{n} \)
- phase shift: \( b \) (shift of \( b \) units to the left)
- vertical shift: \( c \) units upwards
Trigonometric Equations:
If \( \sin x = a \) then \( x = n\pi \pm (-1)^n \arcsin a \), \( n \in \mathbb{Z} \)
If \( \cos x = a \) then \( x = 2n\pi \pm \arccos a \), \( n \in \mathbb{Z} \)
If \( \tan x = a \) then \( x = n\pi \pm \arctan a \), \( n \in \mathbb{Z} \)

CALCULUS
Definition of Derivative: \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

Differentiation and Integration

<table>
<thead>
<tr>
<th>Differentiation Formulae</th>
<th>Integration Formulae</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function</strong></td>
<td><strong>Derivative</strong></td>
</tr>
<tr>
<td>( x^n )</td>
<td>( nx^{n-1} )</td>
</tr>
<tr>
<td>( \sin x )</td>
<td>( \cos x )</td>
</tr>
<tr>
<td>( \cos x )</td>
<td>( -\sin x )</td>
</tr>
<tr>
<td>( \tan x )</td>
<td>( \sec^2 x ) or ( \frac{1}{\cos^2 x} )</td>
</tr>
<tr>
<td>( e^x )</td>
<td>( e^x )</td>
</tr>
<tr>
<td>( \log_e x ) or ( \ln x )</td>
<td>( \frac{1}{x} )</td>
</tr>
<tr>
<td>( f(x).g(x) )</td>
<td>( f(x).g'(x) + f'(x).g(x) )</td>
</tr>
<tr>
<td>( \frac{f(x)}{g(x)} )</td>
<td>( \frac{g(x).f'(x) - f(x).g'(x)}{g(x)^2} )</td>
</tr>
<tr>
<td>( g{f(x)} )</td>
<td>( g'{f(x)}.f'(x) )</td>
</tr>
</tbody>
</table>

PROBABILITY DISTRIBUTIONS

Combinations: \( ^nC_r = \frac{n!}{r!(n-r)!} \) \( n! = n(n-1)(n-2) \cdots 3 \times 2 \times 1 \)

<table>
<thead>
<tr>
<th>Discrete Random Distribution</th>
<th>Binomial Distribution</th>
<th>Hypergeometric Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pr(X=x) )</td>
<td>as table</td>
<td>( \Pr(X=x) = ^nC_r p^x (1-p)^{n-x} )</td>
</tr>
<tr>
<td>Expected Value</td>
<td>( E(X) = \sum(x.\Pr(X=x)) )</td>
<td>( \mu = np )</td>
</tr>
<tr>
<td>Variance</td>
<td>( \sigma^2 = E(X^2) - [E(X)]^2 )</td>
<td>( \sigma^2 = np(1-p) )</td>
</tr>
<tr>
<td>Standard Normal:</td>
<td>( z = \frac{x - \mu}{\sigma} )</td>
<td></td>
</tr>
</tbody>
</table>

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