This was the fourth year where examination consisted of two parts in separate booklets. This was the second year where candidates were able to write during the 15 minute ‘reading time’ but unable to use their calculator. 80 minutes after the end of ‘reading time’ candidates were stopped and the Part 1 booklet was collected. Candidates then had 100 minutes of working time in which they were allowed to use their calculators. Candidates who completed Part 1 of the examination within the first 80 minutes were permitted to start Part 2, but without the aid of calculators.

A team of 15 markers met for the first day of the marking period. A discussion of the examination paper, generally, as well as a question by question analysis was undertaken. Advice relevant to the marking of the questions, as well as any specific concern, was shared. Markers then used this information to mark a sample of scripts. Once common agreement on standards and evidence required was agreed upon, marking in full commenced. A team of 5 markers took responsibility for the marking of Part 1 of the examination. Part 2 was marked by a team of 10 markers working in pairs, each marking a section relating to one of the five assessment criteria.

After the marking was completed the Assessment Panel met and considered the distribution of results and looked at all candidate results that were identified as borderline or an anomaly. The cut-offs for each criterion were:

**Criterion 3 – Function Study –**

A: 24  B: 18  C: 12

**Criterion 4 – Circular Functions –**

A: 23  B: 18  C: 12

**Criterion 5 – Differential Calculus –**

A: 24  B: 18  C: 12

**Criterion 6 – Integral Calculus –**

A: 22  B: 18  C: 11

**Criterion 7 – Probability –**

A: 23  B: 18  C: 11

Future students are reminded that this was the last year of the MTM315109 course and that the MTM315114 course contains differences.

**PART 1**

Overall Part 1 was done well. The most common source of lost marks was poor arithmetic or poor algebraic skills. The markers strongly recommend that students who cannot do arithmetic using fractions, decimals and signed numbers without the aid of their calculator seek additional numeracy practice.
SECTION A – FUNCTION STUDY

Question 1

i) Very, very few answered this question correctly. Many students counted $x^2+9$ as having two additional intercepts.

ii) Mostly well done.

iii) Mostly done well, but some made disturbing errors; multiplying $13\times4$ incorrectly or adding the brackets rather than multiplying.

Question 2

Mostly done very well. Most common mistake was to make calculation errors when finding the gradient.

Question 3

Mostly done well. Most common error was to leave the $x$ in the coefficient.

Question 4

i) Mostly done very well.

ii) Many did not know the correct way to write the domain and range, even though they were correct in their understanding. For example, “all the real numbers except $x=4$” should be written $\mathbb{R}\setminus\{4\}$ . Often the slash went the opposite way or was missed out completely. Various ways of expressing the correct domain were given full marks but this area needs improvement. Students should use union symbol if they wish to list it as two sets.

SECTION B – CIRCULAR FUNCTIONS

Question 5

An easy one mark. Although the one mark was obtained for the unsimplified solution of $\frac{105\pi}{180}$, a disturbing number of students, if they progressed further, were unable to accurately reduce this fraction to its simplest form.
Question 6

On the whole this was quite well done. Full marks were not obtained if the result was not evaluated correctly, and this was very common. Many students were able to determine the result for \( \cos \frac{5\pi}{6} \) and \( \tan \frac{4\pi}{3} \), but could not multiply them correctly.

Question 7

Well done. Amplitude of -3 was a common error.

Question 8

Well done.

Question 9

Many students tried to use the fact that \( \tan \theta = \frac{\sin \theta}{\cos \theta} \) and hence stated that \( \cos \theta = 4 \), rather than using Pythagoras’ Theorem to find the hypotenuse of a right triangle. Others did not recognise that \( \cos \theta \) needed to be positive.

Question 10

This was pretty well done with most students obtaining at least half of the available marks. The most common error by far was the failure to recognise that the sine graph had been reflected and so resulting in \( a = -2 \).

SECTION C – DIFFERENTIAL CALCULUS

Question 11

Generally well done. Some students had a \( \sin x \) term rather than a \( \sin 4x \) term.

Question 12

Many students left the answer in a non-factorised form \( 2(x+2)(x+4)^2 + 2(x+2)^2(x+4) \) rather than \( 4(x+2)(x+4)(x+3) \). A significant number of students did this question by expanding the brackets rather than using the product and chain rules. This requires excessive time, is liable to invoke error and doesn’t readily factorise.
Question 13

The use of the limit in differentiation from 1st principles was generally well done though it sometimes dropped off lines involving \( h \) and sometimes was not placed with the expression in \( h \).

Question 14

The responses were many and varied. Very few students drew a stationary point of inflection (with an instantaneous gradient of 0) but drew a point of inflection where the minimum size of the gradient was about 0.8. Many students realized that they were going to run into trouble satisfying \( g(x) < 0, \; g'(x) > 0 \) for \( x > 0 \) with a polynomial such as \( g(x) = (x-1)(x+3)^3 \) so the right hand end of the graph petered out or simplify disappearing below the \( x \) axis. The asymptotic behaviour necessary to satisfy \( g(x) < 0, \; g'(x) > 0 \) for \( x > 0 \) was not well demonstrated on graphs that were in other respects correct.

\[
(x-1)(x+3)^3
\]

SECTION D – INTEGRAL CALCULUS

Question 15

Most candidates got this question correct. Common errors were in integrating the first term correctly and the factor of \( \frac{1}{4} \) and the omission the constant term.

Question 16

Most candidates got the integration of the expression in this question correct with those that didn’t often missing the constant term. Again, most candidates knew the process required to find the constant but were unable to do the numerical calculation correctly.
**Question 17**

About 50% of candidates got this question correct. There were numerous expressions provided by candidates in correctly answering this question. As the question asked for ‘an’ expression there were many that were more complicated than they needed to be but were given full marks if correct. Common errors were integrating across the points of intersection of the curves, including areas outside the shaded section, and including areas more than once in the given expressions.

Candidates were generally not penalised for imperfect notation or the omission of ‘dx’ in their solutions but this is something the markers would like teachers to address in the future.

**Question 18**

Generally this question was not done well. A number of candidates did not make the link with integration and the area between the curve and the x-axis. Many attempted to find an equation for the given curve. Of those candidates who did attempt to find an estimate of the integral, many added the two areas. Again, many candidates used methods that were not accurate enough to gain them full marks.

**Question 19**

Most candidates were able to get some marks for this question but few gained full marks. Common mistakes were the substitution of the terminals in the incorrect order, confusion with the use of absolute value and the random introduction of absolute value to eliminate negative signs. If candidates didn’t use absolute value they got stumped when they were faced with logs of negative numbers. There were many variations of the correct answer.

**SECTION E – PROBABILITY**

**Question 20**

A large number of students fell back to using trees rather than substituting numbers into a formula. A number of students incorrectly approached it as a binomial (both via formula and tree). For those that did use hypergeometric some used fractional values of D or obtained negative values in their °C, or ended up with n less than r. A common incorrect answer was \( \frac{5}{7} \times \frac{4}{6} \times \frac{2}{5} \). Some simply enumerated the possible permutations – \{1,1,2 / 1,2,1 / 2,1,1 / 1,2,2 / 2,1,2 / 2,2,1 / 2,2,2\} (sometimes also including 1,1,1 in their enumeration) – and concluded the probability was \( \frac{3}{7} \) (or \( \frac{3}{8} \)).

**Question 21**

Many students did not see that 97.5% was the one-tailed version of 95% and tried to use the z-formula. Some muddled 97.5% with 99.865% and used 3 standard deviations. Some did not understand
N(160, 15^2) and used a standard deviation of $\sqrt{15}$ or 15. A few students added a centimetre or two of grace so people would not graze their head – while this was recognised by the markers it is not encouraged. Few students considered whether their answer was valid and doors of height 1.3cm through to 57.85m were provided as answers.

**Question 22**

A decent number of students were able to correctly answer this question. Common incorrect answers included peaks at $n=3$ or $n=5$. Others simply copied the $p=0.35$ version. Some copied and vertically dilated the $p=0.35$ case until the $n=2$ probability was 0.65. A small number draw continuous distributions.

**Question 23**

Most were able to correctly write down the formulae for $E(X)$ and $Var(X)$. Some equated $Var(X)$ to $24^2$ rather than 24. Poor arithmetic and algebraic skills cost many students marks. Common mistakes included simplifying $\frac{24}{30}$ to $\frac{5}{6}$ or making plus/minus mistakes. Many students happily listed values of $p$ outside $[0,1]$ and/or gave negative or fractional values of $n$. Some students left answers unsimplified such as $\frac{30}{0.2}$ or $\frac{30}{6}$. In some cases 30 times 30 was also equal to 90 or rarely 9000.

**Question 24**

Part (i) was generally well done with a significant number of students seeing the symmetry. Part (ii) was very poorly done arithmetically with many students unable to correctly work with decimals. Common mistakes included $(-0.5)^2$ being equated to 2.5, -2.5, 1, -1, -0.25, -0.025, -0.0025. Many students did not square the negative correctly then got a variance (and hence standard deviation) of zero, often with incorrect (but symmetric) arithmetic. Values for variance varied from 400 to 0.00004. For those students that did part (ii) correctly only about a quarter correctly evaluated the square root with common errors including 0.02 and 0.16. Those than got part (ii) wrong did equally terrible things with the square root, the most common operation being simply halving the value under the square root. Very few students attempted to convert the decimals to fractions. Those that did generally got full or close to full marks.

**PART 2**

Students are reminded that questions worth three or more marks require relevant working out. “On my Cas calculator” is **not** working out and should not be stated. Nor should calculator commands be copied verbatim from the screen.
SECTION A - FUNCTION STUDY

Question 25

Generally well done. Many students did not recognise that the dilation in the x-axis affected the horizontal translation, i.e wrote trans 3 left instead of $\frac{3}{2}$. Some also still had the horizontal dilation as a factor 2 instead of $\frac{1}{2}$. Order of dilation/translation sometimes confused. Some students wrote the transformations in matrix form. This was awarded only part marks.

Question 26

Use of log laws by many students was very poor. Many students eliminated $x=-2$ without giving a reason. Note that any accepted reasons had to refer to the logarithm, e.g. can't log a -ve, log(-2) does not exist etc. $x>0$ without reference to the log was not sufficient reasoning (lost $\frac{1}{2}$ mark). As the question stated, “determine algebraically”, no marks were awarded if the students simply entered the equation into their calculator and gave the answer. Also, because the question stated determine algebraically, if students went from $\frac{x^2}{x+3} = 4$ straight to the answer they were not awarded full marks.

Question 27

This is a 4 mark question, meaning working or reasoning must be shown for both parts. As always, calculator instructions are not working.

In part (i) full marks were not awarded if students did not write $y^{-1}$ or refer to the inverse in some way. Part marks were also deducted if there was no working between swapping $x$ and $y$ and stating the inverse function. Please note that relations do not have to be 1:1 to be functions and, as the inverse is a quadratic, the restriction is not there to make the inverse a function, it is there so the domain and range of the inverse match the range and domain of the original function.

In part (ii), the point of intersection needed to be justified, either with a well labelled graph showing why there was only 1 solution, or algebra which showed 2 solutions and a reason why one was eliminated. Part marks were deducted if both co-ordinates were not given. Many graphs were poorly done - no points labelled, curve itself not labelled and some were very poor depictions of a quadratic. It is worth noting that very few students actually made reference to their graph when answering part b, which leaves it to the marker to actually notice the graph and assume that was the reason for the solution given.

Question 28

a) Again, relevant working must be shown for full marks to be awarded. Many students missed the horizontal asymptote of $96 = c$ and were unable to show anything further than substituting in the 2 points given. Many students began by assuming $b = e$, but did not adjust the power. Some
recognised the decaying nature, so changed the general equation to \( R = a \times b^t + c \) or worse \( R = -a \times b^t + c \). Some students used this reasoning to change the sign of values they had found.

b) Carried through from a). However, it's worth noting that a large number of students found solutions to equations which the calculator gave no solution for. Half a mark was deducted if they did not give their answer to 2 decimal places (correctly) as this is what the question specified.

SECTION B - CIRCULAR FUNCTIONS

Question 29

This question was generally well answered. Some common misconceptions included:

- Writing “on CAS calculator” as working out for a problem whiched scored very few marks.

- Students often failed to correctly evaluate \( \arcsin \left( \frac{\sqrt{3}}{2} \right) \)

- Some students obtained part scores because they cannot multiply a fraction by 5!

- In general those students who used the trigonometric equations as noted on the information sheet got completely muddled and scored few marks.

Question 30

The first part of this question was a very difficult with very few students obtaining full marks. The “explain why \( \tan \left( \frac{\pi}{2} + x \right) = -\frac{1}{\tan x} \)” requires more than simply repeating the equation or randomly including cotx. Often the inclusion of a diagram indicating the complimentary angles, or the \( \frac{\pi}{2} \) translation, helped students achieve full marks. Impressive solutions included the concept that \( \tan x \) is in fact the gradient of the line and that \( -\frac{1}{\tan x} \) was the normal.

The second part of the question required students to use the previous result to solve a relatively simple equation. Many failed to use this result and did not achieve the full allotment of marks. Again writing “on Cas calculator” as working out scored few marks.

Question 31

This was a straight forward question with most students obtaining at least 2 marks. Those obtaining the full 4 marks outlined where the four values came from. Some students mistakenly tried to determine \( b \) by substituting in the value of one point from the graph. This approach can produce infinitely many (incorrect) solutions for \( b \), resulting in less than full marks for some students.
Question 32

a) Another explain question! The key to this diagram is the vertical asymptote at θ = 0. Students who mentioned this received the mark.

b) Conceptually this was a straight forward question with multiple correct answers accepted. Many students correctly deduced the value of b (a translation of 90 degrees) and then substituted it into the equation along with one data point to easily obtain the answer and full marks. Many students attempted to use the power of the CAS calculator to solve two simultaneous equations with little success. Others became confused between radians and degrees producing a range of incorrect answers.

SECTION C - DIFFERENTIAL CALCULUS

Question 33

Most students made errors in this question. Many students used interval notation that was incorrect or unconventional. Many students were unclear as to whether the function at the points x=1 and x=3 was differentiable; we accepted any interpretation here as long as students were consistent about which groupings the points were placed in. Many students believed the graph stopped at x = -5 and x = 5, which was clearly not true.

Question 34

Most students used their calculators to find the derivative in this question for full marks. Many students made mistakes in transcribing the answer from their calculator, particularly putting brackets in incorrect places. Many students found the derivative by hand which was pleasing to see. Some students lost marks for claiming that $\ln(x)^2 = 2\ln(x)$.

Question 35

This question was reasonably well done by many students. Classifying the stationary points incorrectly or not attempting to classify them at all were common errors. Many students also forgot to find the y-values of the points. Students need to be reminded that $\frac{d^2y}{dx^2} = 0$ at a point is not sufficient for it to be a point of inflection – using a gradient table was more successful here.

Question 36

Most students had little idea how to solve this question. A number erroneously equated the curve and its derivative to find x. Algebraic errors were common in this question.
Question 37

a) Numerous students did not show sufficient reasoning to gain full marks for this question.

b) Some students did not find an expression for the volume correctly and hence attempted to maximise the wrong function. Many students did not find the derivative correctly. Some students did not consider \( r=4 \) as a solution at all before excluding it for physical reasons. Even though \( r=4 \) was the only physical solution students were still required to show it was a maximum to gain full marks. Students commonly found the correct maximum value for this question.

SECTION D - INTEGRAL CALCULUS

Question 38

Reasonably well done. However, there was an obvious lack of understanding of general properties of integrals.

Question 39

A large number of students ignored the candidate instruction “For questions worth 3 or more marks, you must show relevant working” and just wrote the answer. This gained them 2 marks only. Most students who used their calculator only, forgot to include \( +c \), this cost them \( \frac{1}{2} \) mark.

Question 40

a) Well done. Many candidates gave co-ordinates of the points of intersection instead just the \( x \)-values as asked. This was not penalised.

b) The majority of students were able to give the correct expression for the area enclosed. A significant number of these students did not give an exact answer and/or did not give units. Markers were looking for the correct area expression and the exact answer with units.

Question 41

Again students chose to ignore the candidate instruction “For questions worth 3 or more marks, you must show relevant working” so since this question was worth 5 marks then using the calculator gained them 2 marks only. It was essential that the “hence” was used. Again a significant number of students ignored the “exact value”. This question was very poorly done.
SECTION E - PROBABILITY

Students should note that simply copying out what was entered in the calculator or stating “using CAS” is not appropriate mathematical communication. For example, a student who only wrote \( \text{binomialCDF}(1, 35, 35, 1/38) = 0.607 \) was not awarded full marks for Question 42.

**Question 42**

Students had difficulty interpreting the question. Many recognised that the situation was a binomial distribution with \( n = 35 \) and \( p = \frac{1}{38} \) but were not able to progress further. Common errors were finding the probability of one success (rather than the probability of greater than one success), using the hypergeometric distribution and finding the mean.

**Question 43**

Generally well done. Appropriate working was required for students to gain full marks. Care should be taken not to round off early – for example, some students rounded the z score off to one decimal place which resulted in a different answer.

**Question 44**

Part (a) was done well. Many errors were made in part (b). Students struggled to use the variance formula correctly, found an incorrect value for \( E(Y^2) \) or incorrectly evaluated the solutions of the quadratic. It was sensible to use the calculator to find the solutions. Students needed to record both solutions for \( b \) and state why the negative \( b \) value was rejected.

**Question 45**

Part (a) was worth 3 marks and (b) worth 2 marks. Many students were able to find \( \text{Pr}(X=2) \) in \( ^nC_r \) format but struggled to simplify the factorial terms. It was again sensible to use the calculator to find the solutions in part (b) rather than solve by hand. Students needed to record both possible values for \( n \) and state why the fractional \( n \) value was not a valid solution.
Non–Calculator

Function Study

1)  a) x–intercepts: 2, y–intercepts: (0,−18)
   b) \( f(2) = (2−1)^3(2+2)(2^2+9) = 52 \)

2)  \( y = a \left| x−4 \right| − 3 \)
    \( 5 = a \left| 0−4 \right| − 3 \)
    \( 8 = 4a \)
    \( a = 2 \)
    \( y = 2 \left| x−4 \right| − 3 \)

3)  \( ^5C_2\times(3)^4 \)
    \( = 5\times2\times3^4 \)
    \( = 10\times81 \)
    \( = 810 \)

4)  Domain: \( \mathbb{R} \setminus \{4\} \), Range: \( \mathbb{R} \setminus \{1\} \)

Trigonometry

5)  \( 105 \times \frac{\pi}{180} \)
    \( = \frac{105\pi}{180} \)
    \( = \frac{7\pi}{12} \)

6)  \( \cos^6\frac{\pi}{6} \times \tan^4\frac{\pi}{3} \)
    \( = −\cos^6\frac{\pi}{6} \times \tan^4\frac{\pi}{3} \)
    \( = −\frac{\sqrt{3}}{2} \times \sqrt{3} = −\frac{3}{2} \)

7)  Amplitude: 3, period: \( \frac{2\pi}{4} = \frac{\pi}{2} \)

8)  Quadrants: 3,4, basic angle: \( \frac{\pi}{6} \)
    \( x = \pi + \frac{\pi}{6}, 2\pi − \frac{\pi}{6} \)
    \( = \frac{7\pi}{6}, \frac{11\pi}{6} \)

9)  As \( \pi \leq \theta \leq 2\pi \) and \( \tan \theta < 0 \) then quadrant 4.

10)  \( a = −2, b = \frac{2\pi}{4\pi} = \frac{1}{2}, c = 4 \)

Differential Calculus

11)  \( −4\sin 4x + 10(2x+3)^4 \)

12)  \( g'(x) = 2(x+2)(x+4)^3 + 2(x+2)^2(x+4) \)
    \( = 2(x+2)(x+4)[x+4 + x+2] \)
    \( = 2(x+2)(x+4)(2x+6) \)
    \( = 4(x+2)(x+3)(x+4) \)

13)  \( f'(x) = \lim_{h \to 0} \frac{f(x+h)−f(x)}{h} \)
    \( = \lim_{h \to 0} \frac{(x+h)^2−3(x+h)+4 − (x^2−3x+4)}{h} \)
    \( = \lim_{h \to 0} \frac{x^2+2xh+h^2−3x−3h+4 − x^2+3x−4}{h} \)
    \( = \lim_{h \to 0} \frac{2xh+h^2−3h}{h} \)
    \( = \lim_{h \to 0} 2x+h−3 \)
    \( = 2x−3 \)

14)  Many answers possible. There must be a stationary point of inflection at \((-3,0)\), a minimum at \(x=0\) and must approach an asymptote as \(x \to \infty\) (value of asymptote does not have to be zero). One possible answer follows.
Integral Calculus

15) \( \frac{1}{4} \cos(4x+1) + \frac{7}{2}x^2 + c, \ c \in \mathbb{R} \)

16) \( y = \frac{4x^3}{3} - \frac{3x^2}{2} - 2x + c, \ c \in \mathbb{R} \)
   \[
   1 = -\frac{32}{3} - 6 + 4 + c
   \]
   \[
   c = \frac{41}{3}
   \]
   \( y = \frac{4x^3}{3} - \frac{3x^2}{2} - 2x + \frac{41}{3} \)

17) \[
\int_a^c g(x)dx + \int_a^c g(x) - f(x) \ dx
\]
   Many other expressions are possible, all of which should reduce down to:
   \[
\int_a^e g(x)dx - \int_a^c f(x) \ dx
\]

18) The precise answer is 17.8 so an answer close to this is acceptable. Note that the integral from \( x = -3 \) to 0 is -4.5 and the integral from 0 to 12 is 22.3 so answers of 26.8 or thereabouts are the area between the curve and the axis not the actual integral.

19) \[
\int_{-4}^{-1} \frac{7}{x} \ dx = (-7 \log_e |x|)^{-1}_{-4}
\]
   \[
   = -7 \log_e 4 + 7 \log_e 1
   \]
   \[
   = -7 \log_e 4 + 7 \log_e 2
   \]

Probability

20) \[
\frac{\binom{5}{3} \cdot \binom{2}{2}}{\binom{7}{3}} = \frac{\frac{5!}{3!2!} \cdot \frac{2!}{2!0!}}{\frac{7!}{3!4!}} = \frac{20}{35} = \frac{4}{7}
\]

21) 2.5% will be two standard deviations from the mean so 160 + 2×15 = 190cm.
Calculator Section

Function Study

25) Either: Horizontal dilation/dilation parallel to x-axis by \( \frac{1}{2} \) then horizontal translation \( \frac{3}{2} \) units left. And a vertical translation of 7 (order of vertical translation doesn’t matter).

Or: Horizontal translation 3 units left then horizontal dilation/dilation parallel to x-axis by \( \frac{1}{2} \). And a vertical translation of 7 units up (order of vertical translation doesn’t matter).

Or: Vertical dilation/dilation parallel to y-axis by \( \frac{1}{4} \) then vertical translation of 7 units up. And a horizontal translation of \( \frac{3}{2} \) units left. (order of horizontal translation doesn’t matter).

26) \[2 \log_2 x - \log_2 (x+3) = 2\]
\[\log_2 x^2 - \log_2 (x+3) = 2\]
\[\log_2 x^2 = 2\]
\[x^2 = 4\]
\[x = \pm 2\]
\[x^2 - 4x + 12 = 0\]
\[(x-6)(x+2) = 0\]
\[x = 6\]
\(x \neq -2\) as the log of a negative isn’t real.

27) (i) \[x = 2 + \sqrt{y+4}\]
\[(x-2)^2 = y+4, x \geq 2\]
\[x^2 - 4x + 4 = y+4, x \geq 2\]
\[x^2 - 4x = y, x \geq 2\]
\[x^2 - 4x = x\]
\[x^2 - 5x = 0\]
\[x(x-5) = 0\]
\[x = 5\]
\[x \neq 0\] as it is outside the domain.

(ii) \[x = 2 + \sqrt{y+4}\]
\[(x-2)^2 = y+4, x \geq 2\]
\[x^2 - 4x + 4 = y+4, x \geq 2\]
\[x^2 - 4x = y, x \geq 2\]
\[x^2 - 4x = x\]
\[x^2 - 5x = 0\]
\[x(x-5) = 0\]
\[x = 5\]
\[x \neq 0\] as it is outside the domain.

28) (a) \[c = 96\]
\[160 = ab^0 + 96\]
\[a = 64\]
\[6346 = 64b^{-2.5} + 96\]
\[6250 = 64b^{-2.5}\]
\[\frac{6250}{64} = b^{-2.5}\]
\[b = \frac{4}{25} = 0.16\]

(b) solve \((64(0.16)^x + 96 = 101, x)\) \(\Rightarrow x = 1.40\) (2dp)

Trigonometry

29) \[\sin \left( \frac{x}{2} \right) = -\frac{\sqrt{3}}{2}\]

Quadrant: 3, base angle: \( \frac{\pi}{3}\).

\[\frac{x}{2} = \pi + \frac{\pi}{3}\]
\[x = \frac{20\pi}{3}\]
\[\tan(x + \frac{\pi}{2}) = -\cot x\] (complimentary angle)
\[= -\frac{1}{\tan x}\] (definition of cot)

Solving: \[\tan(x + \frac{\pi}{2}) = \sqrt{3}\]
\[-\frac{1}{\tan x} = \sqrt{3}\]
\[\tan x = -\frac{1}{\sqrt{3}}\]

Quadrants: 2, 4, basic angle: \( \frac{\pi}{6}\).

\[x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}\]
\[x = \frac{5\pi}{6}, \frac{11\pi}{6}\]

31) \[a = 5, d = 2, c = \frac{\pi}{6}\]

\[3 \frac{2}{3} \text{ period } = \pi\]

\[\text{period } = \frac{2\pi}{3}\]
\[b = \frac{2\pi}{3}\]
\[b = 3\]
32) (a) A tan curve is the only trigonometric function which can account for the fact that there is an unlimited/infinity value occurring, e.g. an asymptote.
(b) Asymptote at x=0, zero at x=90 so b=90. Subbing in a value (students only need to use one value, all variations are listed for checking purposes):

<table>
<thead>
<tr>
<th>x</th>
<th>tan(x) + 90°</th>
<th>y = x</th>
<th>y = -x</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>90°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>90°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>90°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Differential Calculus

33) (i) −1<x<1
(ii) x<−1 or 1<x<3
(iii) x>3
(iv) x=−1, x=1, x=3

34) \[ y' = \frac{e^{x} - \log_{e}x - x^{2}e^{x}}{(\log_{e}x)^{2}} \]

35) \[ y' = 3x^{2}e^{x} + x^{3}e^{x} \]

Stationary points when y'=0

<table>
<thead>
<tr>
<th>x</th>
<th>y'</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
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<tr>
<td>-1</td>
<td>0</td>
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<td>10</td>
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So (−3,27e−3) is a minimum and (0,0) is a point of inflection.

36) \[ f'(x) = 4x+3 \quad \text{& gradient of tangent} = -1 \]
\[ 4x+3 = -1 \]
\[ 4x = -4 \]
\[ x = -1 \]

At x=−1 y = 6
\[ f(−1) = 6 = 2−3+a \]
\[ a = 7 \]

37) (a) \[ \frac{96\pi}{2} = \pi r(r+h) \]
\[ 48 = r^2 + rh \]
\[ rh = 48 - r^2 \]
\[ h = \frac{48-r^2}{r} \]

(b) \[ V = \pi r^2 (48-r^2) \]
\[ V = 48\pi r - \pi r^3 \]
\[ V' = 48\pi - 3\pi r^2 \]

Maximum when V'=0
\[ 0 = 48\pi - 3\pi r^2 \]
\[ 3\pi r^2 = 48\pi \]
\[ r^2 = 16 \]
\[ r = 4 \]

Maximum \[ V = \frac{4\pi(48-4^2)}{3} = 128\pi \text{ units}^4 \]

Integral Calculus

38) \[ \int 3f(x) - 4x \, dx \]
\[ -2 \]
\[ = 3 \int f(x) \, dx - \int 4x \, dx \]
\[ -2 \]
\[ = 3 \int f(x) \, dx - 2 \]
\[ = 3(−7) - 2 \]
\[ = 3(−7) - (2x)^3 \]
\[ = -21 - 2x^3 - 2x(-2)^3 \]
\[ = -21 - (18 - 8) \]
\[ = -21 - 10 \]

39) \[ \int (4x+1)^5 + (2e^x-3)^2 + \frac{4}{(x-2)^2} \, dx \]
\[ = \int (4x+1)^5 + 4e^{2x} - 12e^x + 9 + \frac{4}{(x-2)^2} \, dx \]
\[ = \frac{1}{24}(4x+1)^6 + 2e^{2x} - 12e^x + 9x - \frac{4}{(x-2)^2} + c, \ c\in\mathbb{R} \]

40) (a) \[ x = 0, 1, 3 \]

(b) A = \[ \int x^2 + 5x - 12 + 12 \cos \frac{\pi x}{3} \, dx \]
\[ = \int x^2 + 5x - 12 + 12 \cos \frac{\pi x}{3} \, dx \]
\[ = \left( \frac{x^3}{3} + \frac{5x^2}{2} - 12x + \frac{36 \sin \frac{\pi x}{3}}{\pi} - \frac{36 \sin \frac{\pi x}{3}}{3} - \frac{5x^2}{2} + 12x \right) \]
\[ = \left( \frac{x^3}{3} + \frac{5x^2}{2} - 12x + \frac{36 \sqrt{3}}{\pi} \right) - \left( -9 - \frac{45}{2} + 36 + 0 \right) \]
\[ = \left( \frac{3 \sqrt{3}}{\pi} - \frac{83}{6} \right) \]
41) \( \frac{d}{dx} xe^{2x} = e^{2x} + 2xe^{2x} \)

So \( \int e^{2x} + 2xe^{2x} \, dx = xe^{2x} + c, \, c \in \mathbb{R} \)

\( \int e^{2x} \, dx + \int 2xe^{2x} \, dx = xe^{2x} + c \)

\( \int 2xe^{2x} \, dx = xe^{2x} - \int e^{2x} \, dx + c \)

\( \int 2xe^{2x} \, dx = xe^{2x} - \frac{1}{2} e^{2x} + c \)

\( \int xe^{2x} \, dx = \frac{1}{2} xe^{2x} - \frac{1}{4} e^{2x} + c_2, \, c_2 \in \mathbb{R} \)

So \( \int xe^{2x} \, dx = \frac{1}{2} xe^{2x} - \frac{1}{4} e^{2x} \) evaluated from 0 to e

\( \int xe^{2x} \, dx = \frac{1}{2} e^{e+1} - \frac{1}{4} e^{2e} + \frac{1}{4} \) evaluated from 0 to e

\( \int xe^{2x} \, dx = \frac{1}{2} e^{e+1} - \frac{1}{4} e^{2e} + \frac{1}{4} \)

42) \( n = 35, \, p = \frac{1}{38} \)

\( Pr(X \geq 1) = 1 - Pr(X = 0) \)

\( = 1 - \left(1 - \frac{1}{38}\right)^{35} \)

\( = 0.606781 \)

\( = 60.7\% \)

43) \( Pr(Z > z) = 0.1 \)

\( z = 1.28155 \) (from calculator)

\( \frac{1.28155}{\sigma} = \frac{85-47}{\sigma} \)

\( \sigma = \frac{38}{1.28155} \)

\( \sigma = 29.7 \)

44) \( Var(Y) = 0^2 \times a + 2^2 \times b + 4^2 \times 1 + 5^2 \times 2 + 6^2 \times 1 - (0 \times a + 2 \times b + 4 \times 1 + 5 \times 2 + 6 \times 1)^2 \)

\( 3.2 = 4b + 1.6 + 5 + 3.6 - (2b+2)^2 \)

\( 3.2 = 4b + 10.2 - 4b^2 - 8b - 4 \)

\( 4b^2 + 4b - 3 = 0 \)

\( (2b+3)(2b-1) = 0 \)

\( b = 0.5, \) \( b \neq -1.5 \) as \( b \) is a probability.

45) (i)

\( Pr(X=2) = \frac{nC_2 \times c_1^2}{n^2C_3} \)

\( = \frac{n(n-1) \times 2}{2 \times 1 \times n+2(n+1)n} \)

\( = \frac{6n(n-1)}{(n+2)(n+1)n} \)

\( \frac{6(n-1)}{(n+2)(n+1)n} \)

(ii)

\( \frac{6(n-1)}{(n+2)(n+1)n} = 35 \)

\( 35(n-1) = 2(n+2)(n+1) \)

\( 35n - 35 = 2n^2 + 6n + 4 \)

\( 0 = 2n^2 + 29n + 39 \)

\( 0 = (2n + 3)(n - 13) \)

\( n = 13, \, n \neq -1.5 \) as its the number of balls

So 13 balls.
Award Distribution

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Student Distribution (SA or better)

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<tr>
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