This was the third year where examination consisted of two parts in separate booklets. The examination was prepared, checked and finalised before an alteration to examination procedure was announced by the Assessment Authority. The consequence of this announcement was that candidates could commence writing answers in the examination booklets during what was notionally provided as ‘reading time’. This circumstance does not fit well with an examination in which candidates are restricted from using CAS calculator technology in one part of the examination. As with the previous years, calculators were not allowed to be used during the first part of the examination, or the ‘reading time’. After 80 minutes of working time, candidates were stopped and the Part 1 booklet was collected. During the second 100 minutes of working time candidates were allowed to use their calculators. Candidates who completed Part 1 of the examination within the first 80 minutes were permitted to start Part 2, but without the aid of calculators.

A discussion of the examination paper, generally, as well as a question by question analysis was undertaken in both marking centres. Advice relevant to the marking of the questions, as well as any specific concern, was shared. Markers then used this information to mark a sample of scripts. Once common agreement on standards and evidence required was agreed upon, marking in-full commenced. The team of 6 markers took responsibility for the marking of Part 1 of the examination. One marker was principally responsible for each section of part 1, with the sixth marker working as a collaborative marker on all sections of part 1. Part 2 was marked by the team of 10 markers, working in pairs.

After the marking was completed the Assessment Panel met and considered the distribution of results and looked at all candidate results that were identified as borderline or whose results were identified as an anomaly.

PART 1

Section A

Question 1

Generally well done. Most common errors were candidates missing the double root on \((x-1)^2\) or the zero root giving an ‘x’ factor. Candidates were also penalised for writing an expression rather than an equation.

Question 2

Most candidates were able to translate this (although quite a few translated right or up instead of left) and quite a few made a reasonable attempt at the absolute value.
Question 3

The most common mistakes here were due to poor algebra skills, particularly when multiplying by -1. A number of candidates had poor handwriting, causing them to change $2-y$ to $2y$. Many candidates left off the domain, or were confused between a restricted domain and the maximal domain so stated no restrictions existed. A number of candidates tried to expand once they got to $(x-1)(2-y)$ and very few of these were able to get a correct inverse function from there. Penalties were given for candidate who did not express their answer with $g^{-1}(x)$, especially if they wrote $g'(x)$.

Question 4

a) Many candidates were confused by the – sign in front of the x and translated left instead of right. They then received conflicting information when finding x and y intercepts, but rather than correct their graph, simply made the (correct) answers negative. Some did not simplify the y intercept and left it as $\log_2 4+1$ and therefore did not sketch the correct y-intercept of 3.

b) Most mistakes here were caused by either a complete lack of understanding of log laws or very poor linear algebra skills once the hard work was done. A few changed the base of the log to e. Most worrying were candidates who cancelled the x variables in $(4-x)/x$ and ended up with answers like ‘$8=10$, therefore $0=2$, therefore $x=2$’ showing a complete lack of understanding of any sort of algebraic process at all.

Section B

This section was generally well attempted. Candidates were able to gain several marks from ‘seeing’ the answer but often failed to achieve full marks due to a lack of working being shown.

Question 5

Most candidates made use of the formula sheet and were able to set up the conversion but many candidates could not simplify the fractions involved. There is a real need for candidates to practise simple fraction manipulations and simple division. There is too much reliance on calculators.

Question 6

Candidates were mostly able to find the correct exact values but many incorrectly located the appropriate quadrant and consequently used the incorrect signs of the exact values. Some candidates lost part marks as a result of their incorrect simplification of the two fractions and surds. Again, these basic skills should be practised if candidates are weak in these areas. Full
marks were awarded to answers that remained in separate surd fractions. (It was not expected that the fractions to be combined using a common denominator).

**Question 7**

Being a ‘3 mark question’, many candidates failed to provide reasoning for their answers and consequently did not gain full marks.

Most candidates did not recognise the reflection of f(x), but did see the dilation by a factor of 3 (in the y-direction).

In the calculation of the dilation factor in the x-direction, most candidates arbitrarily allocated a scale to the x-axis, most saying that f(x) had a period of $2\pi$. This then gave a value of $b=2$. Whilst this is the correct answer it is not the correct reasoning. Others tried to consider the functions as ‘Acos $n(x)$’ type graphs and went on to determine the value of $n$. Again, this is an incorrect approach. Only a few candidates were able to say that ‘the period of g(x) is half that of f(x) and so $b=2$’

**Question 8**

This question was worth 5 marks and so both parts (a) and (b) required working to support the answers, otherwise full marks could not be allocated.

a) Most candidates correctly found the two heights but did not explain the reasoning for them. Full marks could not be given for just showing the ‘max. (height) = 150m and the min. (height) = -50 m.’ Candidates need to see that this question has a ‘real-life’ scenario and consequently needs the answers to be in terms of the variable asked in the question. In this case, ‘max. height above the pipeline’ and ‘greatest depth below the pipeline.’ (No marks were deducted if the depth was shown as negative 50m. Some candidates attempted to find the max/min by differentiating but most of these candidates were unable to complete the process.

b) The wording of this question was slightly ambiguous and many candidates interpreted the question to mean that they had to find the ‘total length of the pipeline’ rather than the two separate sections of the pipeline (through the mountain, and over the bridge). Consequently these candidates found the period of the function and if they stated that ‘the length of the pipeline is 1 period $\Rightarrow length = 1200m$’, they were given 2 marks. Some candidates found the two lengths by finding the x-coordinates of the max and min and using symmetry. Most others went through the process of finding the x-intercepts and using these values. The most common mistake when using this approach was to correctly find the base angle to be $\frac{\pi}{4}$ but then not to find the correct quadrants for which cos is negative.

i.e. candidates wrote (incorrectly) that $\frac{\pi}{400} = \frac{\pi}{5}$.
Many, however, went on to correctly find the appropriate lengths from this (mistaken) value.

Again, candidates made silly mistakes when manipulating the fractions involved in isolating $x$.

Section C

Question 9

Most candidates used the quotient rule correctly, substituting in the correct initial values. However any subsequent simplification was, in most cases, done poorly with terms within the fraction cancelled incorrectly. There were a large number of candidates who did not attempt any form of simplification of the terms within the fraction; this could have been due to restricted time or lack of confidence in algebra skills.

Some candidates chose to solve the problem using the product rule. This introduced the need to differentiate a composite function, and most were unsuccessful in their attempts.

Question 10

Although this question was conceptually difficult almost all candidates completed it correctly. In fact it was by far the best answered question in this section.

Question 11

a) Many candidates failed to recognise that the product rule was required. A significant number of candidates who did use the product rule correctly made errors in the simplification. The most common error found here was $x \cdot \frac{1}{x} = 0$.

b) Of those candidates whom completed part (a) correctly, most substituted and simplified the expression as required. A few candidates did not know how to simplify $\log_{e} e^x$.

Question 12

a) This was by far the poorest answered question in the section. Most had problems dealing with the fact that the coordinates of the chord were given in function form. Many went into convoluted methods to try and find the gradient. Most candidates, although failing to find a correct solution here, went on to part (c) found and used the gradient of the cord as part of the limiting condition in calculating the gradient of the function at point $A$. I think this shows that the candidates can use the gradient of a function from first principles but have not yet reached that true understanding of what is going on.
b) There were some interesting answers to what is the limiting condition. One that sticks in my mind (and is probably true judging from the answers to this question) was ‘first principles’.

c) Generally answered well and regularly included the correct working for part (a). It was very common for candidates to miss showing the solution for g’(3).

Section D

Question 13

Generally well done. A significant number of candidates missed ‘+c’ or had \( \frac{\sqrt{x}}{2} \) instead of \( 2\sqrt{x} \).

Question 14

Generally well done. Common errors included multiplying by 2 (instead of dividing by 2) and difficulty substituting.

Question 15

Many candidates were able to integrate this successfully.

Solving the quadratic equation seemed to be cause more problems than finding the definite integral.

Interestingly, some candidates saw this as a \( \int (ax+b)^n \) type of problem which made the algebra a little more challenging.

Some candidates worked through the problem successfully only to eliminate \( k = -2 \) as a solution since they believed that the upper terminal must be greater than the lower terminal. This error did not gain full marks.

Question 16

This was a polarising question – candidates either did well or did very poorly.

a) One mark was given for the co-ordinates of A and one mark was given for the definite integral. A common error was using \( e \) as the upper boundary for the area.

b) One mark for correct answer (including follow through from (a)).

c) Quite poorly done. Two marks awarded for correct answers. Some candidates attempted to find \( \int_{-1}^{1} e^x \) as the shaded area. No marks were awarded for using terminals of -1 and 1.
Candidates who recognised that the highlighted area was the combination of a rectangle and the area from part (b) were successful. Candidates who missed this connection struggled with this question.

Section E

There were many, and consistent, errors with operations with fractions and decimals. Candidates gave probabilities that were greater than 1. Probabilities were converted into percentages and ‘chance’ mentioned in answers even though only probabilities were asked for in the questions.

Small calculation errors and non-simplification of fractions was ignored by the markers.

There was some confusion between variance and standard deviation. Formulae were often used incorrectly.

Question 17

The mean was found successfully. Many candidates were unable to work with fractions well enough to get to a final answer for the variance. Many candidates stopped at \( \sigma^2 = \left( \frac{12}{7} \right) \left( \frac{2}{7} \right) \). A single fraction answer was required.

Some candidates went further and found the standard deviation. Some candidates used the Binomial Distribution formulae.

Question 18

Demonstration that an ‘or’ situation exists in this problem was required for any marks to be obtained. There were errors with multiplying and adding decimals. Candidates gave answers greater than 1, such as 6.4. There was no need to give the answer as a percentage.

Question 19

While a correct equation was readily found from the given information, many candidates could not solve the quadratic equation.

From \( p(1 - p) = 0.21 \) many concluded \( p = 0.21 \) and \( p = 0.79 \). Some used the quadratic formula but few obtained correct answers.
Question 20

a) The answer is $k = 9$ not $k = \frac{9}{3v}$. Some candidates worked with $21k$ instead of $21 + k$.

b) Candidates missed the fact that the question asks for ‘at least 2 hours’. Some complicated the question by using some version of the expected value formula. The answer is $\frac{2}{2} \left( \frac{30}{30} \right)$ was accepted) not 66.67% or any variations of this. However, candidates were not penalised for converting the fraction to a percentage.

c) The formula for variance was often used incorrectly. Candidates squared the probability instead of the score. The $[E(X)]^2$ term was sometimes left off. Candidates finished off their working with $\frac{-5}{5}$ when their working clearly did not.

Follow on marks were given when errors had been made in (a) and/or (c) and if the working was correct they were given 2 out of 2 marks.

PART 2

Section A

Function study generally well done.

Question 21

The concept appears to be well understood. A number of calculation errors along the way. However, part marks given where appropriate working provided.

Question 22

Generally well done. Common error was not understanding $f^{-1}$ as an inverse and instead drawing a gradient function. Some candidates had the correct shape for the curve, but reflected in the $y$-axis of the new graph.

Question 23

(a) Generally set up equations correctly and used calculators or algebra to determine solution for $k$. A lot of candidates gave the answer in a more complex form consisting of two terms. A significant number of candidates set up the equations correctly, but didn’t know what to do next.

(b) Most candidates set up required equation to find $D_{30}$. However, as solution was given it was not appropriate to use calculator to determine solution. Candidates expected to provide detailed algebraic reasoning as part of the ‘show’. Candidates who had an
approximate answer in part (a) used this in (b) and obviously could not get exact value required. Clearly more work needed on how to respond to ‘show’ style questions.

**Question 24**

(a) Generally well done, although a number of candidates didn’t list variables \( h, k \) and \( a \) as required.

(b) Difficulty in managing \( f(2x+1) \) and working out the appropriate substitution required. Most candidates were still able to find correct solutions to incorrect substitution, and gain some marks.

**Section B**

Candidates did not consistently work with radians and even switched between degrees and radians within the same question. Rounding was a major issue within this section, with a number of candidates unsure of appropriate level of accuracy. Some incorrectly rounded to 1 significant figure.

**Question 25**

(a) Surprising how many candidates could not complete this correctly, given only required entry into calculator. Suggest ability to control domains or windows in calculator evident here.

(b) Rounding issue common but reasonably well done.

**Question 26**

Generally poorly done with limited understanding of unit circle in relation to \( \tan(\theta) \). Part (c) goes back to ‘show’ issues and what statements required. Direction to use data from part (b) often ignored.

**Question 27**

(a) A number of candidates had good success with this question. However many struggled to label x axis and did not understand notion of a ‘period’ given this was not a regular function. A number used decimals which was deemed OK as exact values not stipulated but issues like \( 6.28 = 6 \) again highlighted inappropriate rounding issues.

(b) (i) Translation to left well done.

(ii) Candidates often missed the negative sign for the translation to right.
Question 28

(a) Generally done OK, but surprising number of candidates could not complete the algebra to write tangent equivalent. A lot of candidates wrote down \( \tan 2x - \frac{\pi}{2} = \frac{1}{\sqrt{3}} \).

(b) Most candidates had a go and gained some level of success. Many gained full marks. It was not sufficient to provide a calculator solution only.

(c) Some candidates did not understand the link between parts (b) and (c) and so solved the equations again, sometimes with different results. Generally, those with good success on the previous part were also successful here.

Section C

Question 29

Most candidates correctly determined expression for the derivative but many did not make the link to the (-1) gradient leading to the appropriate equation to solve. Full marks awarded if solutions just stated.

Question 30

Very few candidates were able to show how quadratic equation developed. However, majority of marks awarded for part (b) as this was the main differential calculus focus. Most correctly solved \( R'(x) = 0 \) giving \( x = 2.5 \). Very few went on to state fee per month as \( 25 - 2.5 = 22.50 \). Some went on unnecessarily to fond the revenue.

Question 31

(a) Majority of candidates understood the shape of the graph. Markers placed importance to open circles at \( x = 2 \) and 3 to highlight discontinuity and roughly equal spacing above and below the x axis of the horizontal lines to highlight same gradient magnitudes. Markers did not take into consideration whether open or closed circles drawn at \( x = 1 \) and 4.

(b) Many candidates found this question straight forward. Markers looking for clear interpretation of both asymptotes and position of each curve.

Question 32

(a) Poorly answered especially in terms of candidates developing reasoned statements that were easy to follow for both the swim and walking legs. Many used Pythagoras
working instead of recognising $\cos \theta = \frac{adj}{hyp}$. There were limited explanations linking distances to times.

(b) A number of candidates completed this question with solid reasoning. However, there were a large number unable to derive the $T(\theta)$ expression with possibly the $\theta$ and $b$ variables compounding difficulties throughout differentiation and solution. Some differentiated with respect to $b$ rather than $\theta$. Some used solutions outside first quadrant showing no appreciation for situation set up in diagram. Justification, was mainly tackled using gradient tables with some successfully graphing or using second derivative techniques. However, a number did not interpret what was in their table that should have led to a ‘maximum’ conclusion.

(c) Many candidates struggle with substitution and calculation. Whatever angle found in (b) could be used with no penalty.

Section D

Question 33

Full marks were given for correct solution. Many who showed working were obviously struggling with the basic concept requiring the splitting up the integral first. Integration errors even with $\int^3_1 2x$ and $\int^3_1 4dx$ cropping up as part of working.

Question 34

(a) Most recognised $P = \int \frac{dP}{dt}$ but setting up of initial reasoned statements were either poor or completely lacking. Many candidates stopped at $P = 24e^{0.03t}$ and did not consider constant. $t=1$ when $P=27$, $c=27$ or $P=27\ 000\ 000$ were some common errors associated with calculation of constant. Majority of marks awarded for this part.

(b) Needed to determine $t=42$ and then correctly substitute into their expression from (a) for full marks.

Question 35

(a) Most candidates shaded the correction section of graph.

(b) Markers looking for statement of equation to solve before stating of values for $a$ and $b$. Many neglected to determine $y=4$ to complete ‘point’ requirement of question. Too many just putting down $(1,4)$ and $(3,4)$ with no supporting evidence. Some spent too long with lengthy algebra solutions that were often incorrect.

(c) Markers again required full definite integral statement before providing area. Some gave 2.33 instead of exact value as required and again lengthy algebra undertaken with many errors. Even mark distributions between (b) and (c) awarded.
Question 36

(a) Full marks not awarded for candidates just stating answer and no working relating to ‘chain rule’.
(b) Most candidates had an understanding of integration by recognition but again the setting out of reasoned stepwise statements as part of the ‘show’ was often poor. Candidates would either make large steps that were not explained or incorrectly come up with final answer with incorrect conclusions to their algebra. Most marks allocated to part (b).

Section E

Question 37

(a) Very few candidates understood what the term ‘sample space’ meant.
(b) Generally handled well

Question 38

As no ‘scaffolding’ within this question, candidates often struggled to lay out reasoned statements leading to conclusion. Most recognised as binomial application with fewer making the $P(X \geq 1) = 1 - P(X = 0)$. Candidates could either solve as equality or inequality before reaching final conclusion. Need to show what equation was solved lead to $N=15.93$. Sometimes candidates left this out and just stated $N=16$ without justifying this as interpretation. Others lost a mark if smallest integer not given. Overall, this question was either poorly or well done.

Question 39

(a) $P(X = 0)$ and $P(X = 1)$ values generally well handled with calculator. However, a number did not understand difference between $\leq$ and $\geq$, interpreting $P(X \leq 1)$ as $P(X \geq 1)$ instead.
(b) Most understood the improvement with approximation as $N$ gets large in relation to $n$. Most candidates made suitable generic statements in regards to this. However, very few received full marks for this part as they neglected to draw in any of probabilities just calculated into explanation.
Question 40

Question was handled really well by most candidates who attempted it. The ‘scaffolding’ obviously structured reasoning.

(a) $z$ values provided mostly correct. Some credit given to candidates who set up statements and/or diagram if they were unable to state values.

(b) Needed to state simultaneous equations. No marks lost if equation in $z = \frac{x - \mu}{\sigma}$ instead of $z\sigma + \mu = x$ format. Some did not make link between $x$ and the upper and lower values. Again, some candidates became locked into lengthy algebra, whereas calculator assistance at this stage acceptable.

(c) Markers looking for appropriate probability statement or diagram leading to $P=0.1736$ before final percentage conclusion.
Section A

Answer ALL questions in this section.

This section assesses Criterion 3.

Question 1  
(2 marks)

Determine a possible equation for the polynomial function, \( y = P(x) \), graphed opposite. Express your answer as a product of linear factors.

\[ y = x(x+2)(x-1)^2 \]

Question 2  
(2 marks)

On the grid below, which gives the graph of \( y = f(x) \), sketch the graph of \( y = |f(x+2)| \).

Section A continues opposite.
Section A (continued)

Question 3

Assuming that the inverse function exists, determine the inverse function \( g^{-1}(x) \) of the function \( g(x) = 1 - \frac{3}{2-x}; x \neq 2 \).

State both the equation and any domain restrictions.

Let \( y = 1 - \frac{3}{2-x} \)

Interchange \( x \) and \( y \) for inverse.

\[ x = 1 - \frac{3}{2-y} \]

\[ x - 1 = -\frac{3}{2-y} \]

\[ x - 1 = -\frac{3}{2-y} \]

\[ 2-y = -\frac{3}{x-1} \]

\[ y = 2 + \frac{3}{x-1} \]

\[ g^{-1}(x) = 2 + \frac{3}{x-1}; x \neq 1, x \in \mathbb{R} \]
Section A (continued)

Question 4

The graph below is of the function \( f(x) = \log_2(x) \).

(a) On the same axes, sketch the function \( g(x) = \log_2(4-x) + 1 \). Clearly label the asymptote and axes intercepts.

\[
g(0) = \log_2(4) + 1 = 3 \\
g(x) = 0 \text{ when } 0 = \log_2(4-x) + 1 \\
\therefore \frac{3}{2} = 4 - x \\
\therefore x = \frac{7}{2}
\]

(b) Solve the equation \( \log_2(x) = \log_2(4-x) + 1 \) and hence determine the coordinates of the point of intersection of \( f(x) \) and \( g(x) \). Express answers as exact values.

\[
\log_2 x = \log_2(4-x) + \log_2 2 \\
\therefore \log_2 x = \log_2 2(4-x) \\
\therefore x = 6 - 2x \\
\therefore 3x = 6 \\
\therefore x = 2 \\
\therefore \text{Pt of intersection is } \left( \frac{5}{3}, \log_2 \frac{5}{3} \right)
\]
Section B

Answer ALL questions in this section.

This section assesses Criterion 4.

Question 5

(a) Convert $\frac{3\pi}{5}$ radians into degrees.

\[108^\circ\]

(b) Convert $18^\circ$ into radians.

\[\frac{\pi}{10}\]

Question 6

Evaluate $\tan\frac{5\pi}{6} + \cos\frac{-\pi}{4}$.

Express answer as an exact value.

\[= \tan\left(\frac{\pi}{2} - \frac{\pi}{3}\right) + \cos\left(-\frac{\pi}{4}\right)\]

\[= -\tan\frac{\pi}{6} + \cos\left(\frac{\pi}{4}\right)\]

\[= -\frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}\]

\[= \frac{3\sqrt{2} - 2\sqrt{3}}{6}\]

Section B continues opposite.
Section B (continued)

Question 7

The diagram below shows the graphs of two circular functions $f(x)$ and $g(x)$.

The function $y = f(x)$ can be transformed into the function $y = g(x)$.

Given $g(x) = a \ f\{b(x)\}$, state the values of $a$ and $b$.

Transformations of $f(x)$

Dilations: factor 3 in the y direction

factor $\frac{1}{2}$ in the x direction

Reflection: in the x-axis

\[ \therefore a = -3, \quad b = 2 \]

Section B continues over the page.
Section B (continued)

Question 8

An engineering company wants to build a new horizontal pipeline which will start by passing through a mountain and finish at the end of a bridge which spans a valley.

The cross-section shows the surface of the land (mountain and valley) and the proposed route (tunnel and bridge). The height of the land above or below the pipeline is modelled by the function \[ H = 100 \cos \left( \frac{\pi (x - 400)}{600} \right) + 50 \], where \( H \) is the height in metres above or below the pipeline and \( x \) is the distance in metres from the axes origin, 0.

(a) What is the greatest height of the mountain directly above the pipeline and the greatest depth of the valley directly below the bridge?

Let \( X = \frac{\pi (x - 400)}{600} \)

\[ -1 \leq \cos (X) \leq 1 \]

\[ -100 \leq 100 \cos (X) \leq 100 \]

\[ -50 \leq 100 \cos (X) + 50 \leq 150 \]

\[ \therefore \text{ Mountain peak is } 150 \text{ m above } \]

\[ \text{ Valley is maximum } 50 \text{ m deep } \]

Section B continues opposite.
Question 8 (continued)

(b) Determine the length of the pipeline section through the mountain and the span of the bridge.

\[
\text{Period is } \frac{2\pi}{x} = 1200
\]

\[1200 \cos \left( \frac{\pi (x - 400)}{600} \right) + 50 = 0\]

\[\therefore \cos \left( \frac{\pi (x - 400)}{600} \right) = -\frac{1}{2}\]

\[\frac{\pi (x - 400)}{600} = \frac{2\pi}{3}\]

\[x - 400 = 400\]

\[x = 800\]

\[\therefore \text{Tunnel is 800 m long}\]

\[\text{and bridge span is 400 m long}\]
Section C

Answer ALL questions in this section.

This section assesses Criterion 5.

Question 9 (2 marks)

Differentiate \( \frac{x^4}{\cos x} \) with respect to \( x \).

\[
\frac{d}{dx} \left( \frac{x^4}{\cos x} \right) = \frac{\cos x \cdot (4x^3) - x^4 \cdot (-\sin x)}{\cos^2 x}
\]

\[
= \frac{4x^3 \cos x + x^4 \sin x}{\cos^2 x}
\]

Question 10 (2 marks)

Determine an expression for \( g'(x) \) in terms of \( h'(x) \) and \( h(x) \), given that \( g(x) = \sqrt{4 + h(x)} \).

\[
g'(x) = \frac{1}{2} \cdot \left(4 + h(x)\right)^{-\frac{1}{2}} \cdot h'(x)
\]

\[
= \frac{h'(x)}{2 \sqrt{4 + h(x)}}
\]

Section C continues opposite.
Section C (continued)

Question 11

Given that \( y = x \log_e x \),

(a) determine an expression for \( \frac{dy}{dx} \)

\[
\frac{dy}{dx} = x \cdot \frac{1}{x} + \log_e x \cdot 1
\]

\[
= 1 + \log_e x
\]

(b) hence determine the rate of change of \( y \) when \( x = e^2 \).

Write your answer in simplest form.

\[
\text{at } x = e^2, \quad \frac{dy}{dx} = 1 + \log_e (e^2)
\]

\[
= 1 + 2 \log_e e
\]

\[
= 3
\]

\( \therefore \) \( f \) is increasing at a rate of 3 units per unit \( x \).

Section C continues over the page.
Section C (continued)

Question 12

(a) Determine a fully simplified expression for the **gradient** of the line segment through \(AB\) on the curve \(g(x) = 5 + \pi x^2\) as shown in the diagram above.

\[
\text{Gradient} = \frac{g(x+h) - g(x)}{(x+h) - x}
\]

\[
= \frac{5 + \pi (x+h)^2 - (5 + \pi x^2)}{x+h - x}
\]

\[
= \frac{5 + \pi x^2 + 2\pi x h + \pi h^2 - 5 - \pi x^2}{h}
\]

\[
= \frac{\pi h (2x+h)}{h}
\]

\[
= \pi (2x+h)
\]

Question 12 continues opposite.
Question 12 (continued)

(b) What is the limiting condition that must be applied to determine an expression for $g'(x)$?

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

(c) Apply this limiting condition to your simplified expression from (a), to determine by first principles an expression for $g'(x)$ and hence an exact value for $g'(3)$.

$$g'(x) = \lim_{h \to 0} \frac{x(2x + h)}{h}$$

$$= \frac{2x}{1}$$

$$\therefore g'(3) = 6\pi$$
Section D

Answer ALL questions in this section.

This section assesses Criterion 6.

Question 13  (2 marks)

Determine the indefinite integral of $\frac{2}{7} - \frac{1}{\sqrt{x}}$ with respect to $x$.

$$\int \left( \frac{2}{7} - \frac{1}{\sqrt{x}} \right) \, dx = \int \left( \frac{2}{7} - x^{-\frac{1}{2}} \right) \, dx$$

$$= \frac{2}{7}x - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{7}x - 2\sqrt{x} + C$$

Question 14  (2 marks)

Find the exact value of the definite integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (4 \cos 2x) \, dx$.

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (4 \cos 2x) \, dx = \left[ 4 \sin 2x \right]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$$

$$= 2 \left( \sin \frac{\pi}{2} - \sin \frac{\pi}{6} \right)$$

$$= 2 \left( 1 - \frac{1}{2} \right)$$

$$= 1$$

Section D continues opposite.
Section D (continued)

Question 15

Solve for $k$: $\int_{0}^{k} (2x+1) \, dx = 2$.

\[
\left[ \frac{x^2+x}{2} \right]^{k}_{0} = 2
\]

\[ k^2 + k - (0+0) = 2 \]

\[ k^2 + k - 2 = 0 \]

\[ (k+2)(k-1) = 0 \]

\[ k = -2 \quad \text{or} \quad k = 1 \]
Section D (continued)

Question 16

The shape below results from a series of reflections across the axes of the function \( y = e^x \) over the domain \([0, 1]\).

![Graph of the function](image)

(a) State the co-ordinates of the point A and hence give a definite integral that represents the area of the shaded section.

\[ A \text{ is } (1, e) \quad \text{Area shaded} = \int_0^1 e^x \, dx \]

(b) Determine an exact value for the shaded area above.

\[ \text{Area shaded} = [e^x]_0^1 \]

\[ = e^1 - e^0 \]

\[ = e - 1 \text{ units}^2 \]

Question 16 continues opposite.
Question 16 (continued)

(c) Hence determine an exact value for the area highlighted below.

\[ \text{Region shaded has area } e - 1 \text{ units}^2 \]

\[ \therefore \text{ Dark shaded region has area} \]

\[ 2e \times 1 - 2(e - 1) \]

\[ = 2e - 2e + 2 \]

\[ = 2 \text{ units}^2 \]
Answer ALL questions in this section.

This section assesses Criterion 7.

Question 17  
(2 marks)

The probability of a single outcome from a hypergeometric distribution is:

\[ \Pr(X = 2) = \binom{4}{2} \frac{3}{C_1} \binom{3}{C_3} \]

Determine exact values for both the mean and variance of the distribution.

\[ X \sim H_g(N, D, n) \quad \mu = \frac{nD}{N} \quad \sigma^2 = \frac{nD}{N} \left(1 - \frac{D}{N} \right) \left( \frac{N-n}{N-1} \right) \]

\[ X \sim H_g(7, 4, 3) \quad \mu = \frac{3 \times 4}{7} = \frac{12}{7} \quad \sigma^2 = \frac{12}{7} \left(1 - \frac{4}{7} \right) \left( \frac{7-3}{7-1} \right) \]

\[ \mu = \frac{12}{7} \quad \sigma^2 = \frac{12}{7} \left( \frac{3}{4} \right) \left( \frac{4}{6} \right) \]

\[ \mu = \frac{24}{49} \]

Question 18
(2 marks)

The probability of a flight from Hobart to Queenstown departing on time, when the weather is fine, is 0.7. If the weather is not fine, the probability of a departure on time is 0.6. In May, the probability of a fine day in Hobart is 0.4.

Determine the probability that, on a particular day in May, the flight will depart on time.  
*Hint: a tree diagram may help.*

\[ P(\text{on time}) = P(\text{fine and on time}) + P(\text{not fine and on time}) \]

\[ = 0.4 \times 0.7 \quad + \quad 0.6 \times 0.6 \]

\[ = 0.28 \quad + \quad 0.36 \]

\[ = 0.64 \]

Section E continues opposite.
Section E (continued)

Question 19

A binomial distribution consists of 100 trials. The variance of the number of successes is 21.

Determine the two possible values for the probability of success on each trial.

\[ X \sim B(100, p) \]

\[ \sigma^2 = np(1-p) \]

\[ 21 = 100p(1-p) \]

\[ 100p^2 - 100p + 21 = 0 \]

\[ (10p - 7)(10p - 3) = 0 \]

\[ p = 0.7 \quad \text{or} \quad p = 0.3 \]
Section E (continued)

Question 20

(5 marks)

Over a 30-day period, Bruce recorded the number of hours he spent kayaking each day. The distribution of these results is shown in the table below.

<table>
<thead>
<tr>
<th>Number of hours, $x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of days on which Bruce spent $x$ hours kayaking</td>
<td>$\frac{1}{30}$</td>
<td>$\frac{9}{30}$</td>
<td>$\frac{k}{30}$</td>
<td>$\frac{11}{30}$</td>
</tr>
</tbody>
</table>

(a) Determine the value of $k$.


(b) Hence find the probability that, on a given day in the 30-day period, Bruce kayaked for at least two hours.

$$P(X \geq 2) = P(X = 2) + P(X = 3)$$

$$= \frac{9}{30} + \frac{11}{30} = \frac{20}{30} = \frac{2}{3}$$

(c) Find the mean number of hours Bruce spent kayaking per day during this period.

$$E(X) = 0 \times \frac{1}{30} + 1 \times \frac{9}{30} + 2 \times \frac{9}{30} + 3 \times \frac{11}{30}$$

$$= \frac{60}{30}$$

$$= 2 \text{ hours}$$

(d) Hence show that the variance for this distribution is $\frac{4}{5}$.

$$E(X^2) = 0^2 \times \frac{1}{30} + 1^2 \times \frac{9}{30} + 2^2 \times \frac{9}{30} + 3^2 \times \frac{11}{30}$$

$$= \frac{144}{30}$$

$$\therefore \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{144}{30} - \left(\frac{2}{3}\right)^2$$

$$= \frac{24}{30}$$

$$= \frac{4}{5}$$
Answer ALL questions in this section.

This section assesses Criterion 3.

Question 21
(2 marks)

Given \( f(x) = 2x^2 - 3x - 4 \) and \( g(x) = 3\sqrt{x} \),

(a) evaluate \( f\{g(4)\} \).

\[
\begin{align*}
  g(4) &= 3\sqrt{4} \\
  f\{g(4)\} &= 2(6)^2 - 3(6) - 4 \\
  &= 60
\end{align*}
\]

(b) evaluate \( g\{f(4)\} \).

\[
\begin{align*}
  f(4) &= 2(4)^2 - 3(4) - 4 \\
  g\{f(4)\} &= g(16) \\
  &= 3\sqrt{16} \\
  &= 12
\end{align*}
\]

Question 22
(4 marks)

The graph of the function \( y = f(x) \) is shown below.

On the axes to the right, sketch a graph of its inverse function, \( y = f^{-1}(x) \).

Label the asymptotes and the image of point A.
Section A (continued)

Question 23

In the first five years after planting, the growth of the trunk of a particular species of tree can be modelled by the equation $D = D_0 \cdot 10^{k t}$, where $t$ is the time in years after planting, $D$ is the diameter of the trunk in centimetres and $D_0$ is the diameter at the time of planting.

The diameter after one year is 50 cm and after three years is 70 cm.

(a) Calculate the exact value of the constant $k$.

\[
\begin{align*}
50 &= D_0 \cdot 10^{k(1)} \\
70 &= D_0 \cdot 10^{k(3)}
\end{align*}
\]

\[
\frac{70}{50} = 10^{2k}
\]

\[
2k = \log_{10} \left(\frac{7}{5}\right)
\]

\[
k = \frac{1}{2} \log_{10} \left(\frac{7}{5}\right)
\]

(b) Hence show that $D_0$ equals $\frac{50 \sqrt{35}}{7}$.

\[
k = \log_{10} \left(\frac{7}{5}\right)^{\frac{1}{2}} = \log_{10} \sqrt{\frac{7}{5}}
\]

\[
\text{Sub in } 0 \quad 50 = D_0 \cdot 10^{\log_{10} \sqrt{\frac{7}{5}}}
\]

\[
\therefore \quad 50 = D_0 \cdot \sqrt{\frac{7}{5}}
\]

\[
D_0 = \frac{50}{\sqrt{\frac{7}{5}}}
\]

\[
= \frac{50 \sqrt{5}}{\sqrt{7}}
\]

\[
= \frac{50 \sqrt{35}}{7}
\]
Section A (continued)

Question 24

A function is of the form \( f(x) = \frac{a}{(x-h)^2} + k \).

(a) Given that \( x = 3 \) and \( y = 5 \) are asymptotes and that \((2, 3)\) is a point on the graph of the function, determine values for the constants \( a, h \) and \( k \).

Using asymptote information

\[
f(x) = \frac{a}{(x-3)^2} + 5
\]

\[
f(2) = 3 \quad \therefore \quad 3 = \frac{a}{(2-3)^2} + 5
\]

\[
-2 = \frac{a}{(-1)^2} \quad \therefore \quad a = -2, \quad h = 3, \quad k = 6
\]

(b) Hence determine fully simplified, exact value solutions of the equation \( f(2x+1) = \frac{9}{2} \).

\[
f(x) = \frac{a}{(x-3)^2} + 5
\]

\[
f(2x+1) = \frac{-2}{(2x+1-3)^2} + 5
\]

\[
= \frac{-2}{(2(x-1))^2} + 5
\]

\[
= \frac{-1}{2(x-1)^2} + 5
\]

\[
f(2x+1) = \frac{9}{2} \quad \therefore \quad \frac{9}{2} = \frac{-1}{2(x-1)^2} + 5
\]

\[
\therefore \quad \frac{-1}{2} = \frac{-1}{2(x-1)^2}
\]

\[
\therefore \quad (x-1)^2 = 1
\]

\[
x-1 = \pm 1
\]

\[
x = 1 \pm 1
\]

\[
x = 2 \text{ or } -2
\]
Section B

Answer ALL questions in this section.

This section assesses Criterion 4.

Question 25

(a) State the number of solutions of the equation \( \sin(2\pi x) = \frac{1}{4} \) in \([-1, \frac{1}{2}]\).

\[ \text{4 solutions} \]

(b) Write down one of these solutions.

\[ x = -0.96 \ldots, x = 0.04 \ldots, x = -0.54 \ldots, x = 0.46 \ldots \]

\[ (+2 \text{ d.p.}) \]

Question 26

The diagram below shows the first quadrant of a unit circle with \( \angle COD = \theta \).

OCD and OAB are similar right angled triangles and \( AB = 2.246 \) units

(a) Determine \( \theta \) to 3 decimal places.

\[ \tan \theta = \frac{2.246}{1} \]

\[ \therefore \theta = 66.000^\circ \]

(b) Determine to 3 decimal places:

(i) \( \overline{OD} \cos \theta = \frac{OD}{AB} \)

\[ \therefore \overline{OD} = 0.407 \]

(ii) \( \overline{DC} \sin \theta = \frac{DC}{AB} \)

\[ \therefore \overline{DC} = 0.914 \]

(c) Use the answers from (b) to show \( \sin^2 \theta + \cos^2 \theta = 1 \).

\[ \therefore \]

Remember: Using answers to 3 decimal places will not produce the exact value.

\[ \sin^2 \theta + \cos^2 \theta \]

\[ = (0.914)^2 + (0.407)^2 \]

\[ = 1.0001 \]

\[ = 1^\circ \]

Section B continues opposite.
Section B (continued)

Question 27

Below is a graph of the function \( f(x) = \sin(x) + \sin(2x) \) over one complete period. Use your calculator to assist with answering the following questions. You may provide answers without giving reasoning.

(a) Label the \( x \) scale on the diagram above and state the period for this function.

\[ \text{Period: } 2\pi \]

(b) The function below results from a translation of the form \( y = f(x + b) \) of the above graph.

(i) State the value of \( b \) that produces this graph by the smallest translation to the left.

\[ b = \frac{2\pi}{3} \]

(ii) State the value of \( b \) that produces this graph by the smallest translation to the right.

\[ b = -\frac{4\pi}{3} \]

Section B continues over the page.
Section B (continued)

Question 28

(a) Express the equation \( \sqrt{3} \cos \left(2x - \frac{\pi}{2}\right) = \sin \left(2x - \frac{\pi}{2}\right) \) in the form \( \tan(ax - b) = c \).

\[
\sqrt{3} = \frac{\sin \left(2x - \frac{\pi}{2}\right)}{\cos \left(2x - \frac{\pi}{2}\right)}
\]

\[
\therefore \tan \left(2x - \frac{\pi}{2}\right) = \sqrt{3}
\]

(b) Hence, determine the exact value solutions of \( \sqrt{3} \cos \left(2x - \frac{\pi}{2}\right) = \sin \left(2x - \frac{\pi}{2}\right) \) over the interval \([0, \pi]\).

\[
\tan \left(2x - \frac{\pi}{2}\right) = \sqrt{3}
\]

\[
\therefore 2x - \frac{\pi}{2} = n\pi + \arctan \sqrt{3}
\]

\[
= n\pi + \frac{\pi}{3}
\]

\[
\therefore 2x = n\pi + \frac{5\pi}{3} + \frac{\pi}{2}
\]

\[
= n\pi + \frac{5\pi}{6}
\]

\[
\therefore x = \frac{n\pi + \frac{5\pi}{6}}{2}
\]

If \( n = -1 \) \( x = -\frac{\pi}{2} + \frac{5\pi}{12} \)

\[= -ve \quad \therefore \text{not in } [0, \pi]\]

\( n = 0 \) \( x = \frac{5\pi}{12} \)

\( n = 1 \) \( x = \frac{11\pi}{12} \)

\[
\frac{11\pi}{12}
\]

Question 28 continues opposite.
Question 28 (continued)

(c) Using solutions from (b), determine the **exact values** of the coordinates of the points of
intersection of \( f(x) = \sqrt{3} \cos \left( 2x - \frac{\pi}{2} \right) \) and \( g(x) = \sin \left( 2x - \frac{\pi}{2} \right) \) over the
domain \([0, \pi]\).

\[
\begin{align*}
f(x) = g(x) & \quad \text{when} \quad \tan \left( 2x - \frac{\pi}{2} \right) = \sqrt{3} \\
& \quad \text{when} \quad x = \frac{5\pi}{12} \quad \text{and} \quad x = \frac{11\pi}{12}.
\end{align*}
\]

\[
\begin{align*}
g \left( \frac{5\pi}{12} \right) & = \sin \left( \frac{10\pi}{12} - \frac{5\pi}{12} \right) \\
& = \sin \left( \frac{\pi}{2} \right) \\
& = \frac{\sqrt{3}}{2}.
\end{align*}
\]

\[
\begin{align*}
g \left( \frac{11\pi}{12} \right) & = \sin \left( \frac{22\pi}{12} - \frac{6\pi}{12} \right) \\
& = \sin \left( \frac{16\pi}{12} \right) \\
& = -\sin \left( \frac{\pi}{3} \right) \\
& = -\frac{\sqrt{3}}{2}.
\end{align*}
\]

\[
\begin{align*}
\text{Pts \ of \ intersection \ are} & \\
\left( \frac{5\pi}{12}, \frac{\sqrt{3}}{2} \right) & + \left( \frac{11\pi}{12}, -\frac{\sqrt{3}}{2} \right).
\end{align*}
\]
Section C

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

**Question 29**

Find the $x$-coordinates of the points on the curve $y = x^3 - 2x^2 + 5$ where the tangents to the curve are parallel to the line $y = 7 - x$.

\[
\frac{dy}{dx} = 3x^2 - 4x \quad \quad \quad \quad 3x^2 - 4x + 1 = 0
\]

\[
(3x - 1)(x - 1) = 0
\]

\[
\therefore \quad y = 7 - x \quad \quad \quad \quad x = \frac{1}{3}, \quad x = 1
\]

\[
\therefore \quad 3x^2 - 4x = -1
\]

**Question 30**

A TV satellite company has 10 000 customers each paying $25 per month. Research has shown the company is likely to gain 500 more customers for every $1 the monthly fee drops.

(a) Show that the revenue per month is $R = -500x^2 + 2500x + 250000$, where $x$ represents the drop in the monthly fee in dollars.

\[
\text{Revenue} = \text{No. of Customers} \times \text{Cost}
\]

\[
= (10000 + 500x)(25 - x)
\]

\[
= -500x^2 + 2500x + 250000
\]

$1$ drops in fee

(b) Apply differential calculus techniques to determine the fee per month that will provide maximum revenue. There is no need to justify that it is a maximum.

\[
\frac{dR}{dx} = -1000x + 2500
\]

\[
\max \text{ when } \frac{dR}{dx} = 0 \quad \therefore \quad 1000x = 2500 \quad \therefore \quad x = 2.5
\]

\[
\therefore \quad \text{Monthly fee for max. revenue is } $22.50
\]

Section C continues opposite.
Section C (continued)

Question 31

Use the axes provided on the right to sketch the gradient functions for both \( y = f(x) \) and \( y = g(x) \) shown below.

Highlight any discontinuities by clearly marking points or asymptotes on the sketches.

(a)

(b)
Section C (continued)

Question 32 (6 marks)

Veronica Smith is at point A on the edge of a circular lake of diameter \( b \) metres.

She swims to B at 1 m/s and then walks at 2 m/s along the lake’s edge to C where she meets Tabitha Jones.

\( \text{ABC is a right-angled triangle, angle } BAC = \theta \text{ and angle } BOC = 2\theta. \) The length of the arc BC equals the length of the radius multiplied by the angle subtended at the centre by the arc.

(a) Show that the time taken, \( T \) (seconds), for Veronica to swim from A to B and then walk from B to C to meet Tabitha Jones is given by:

\[
T(\theta) = b \cos \theta + \frac{b \theta}{2}
\]

\[
\cos \theta = \frac{AB}{b}
\]

\[
\therefore AB = b \cos \theta \quad BC = \frac{b \theta}{2}
\]

\[
\text{Total time} = \frac{\text{dist AB}}{\text{speed AB}} + \frac{\text{dist BC}}{\text{speed BC}}
\]

\[
= b \cos \theta + \frac{b \theta}{2}
\]

Question 32 continues opposite.
Question 32 (continued)

(b) Determine an expression for $T'(\theta)$ and use this derivative to calculate an exact value for $\theta$ where a stationary point exists. State the nature of this stationary point and justify your answer.

$$T'(\theta) = -b \sin \theta + \frac{b}{2}$$

Max/min occurs when $T'(\theta) = 0$

$$b \sin \theta = \frac{b}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$$T'(\theta) = \frac{b}{2}$$

$$T'(\frac{\pi}{6}) = -b + \frac{b}{2}$$

$$= +ve$$

$$= -ve$$

$$\therefore$$ As $\theta$ increases through $\frac{\pi}{6}$

$$T'(\theta)$$ changes $+ \rightarrow -$

$$\Rightarrow$$ Maximum when $\theta = \frac{\pi}{6}$

$$T(\frac{\pi}{6}) = b \cos(\frac{\pi}{6}) + b \frac{\pi}{6}$$

$$= \frac{\sqrt{3}b}{2} + \frac{\pi b}{12}$$

(c) Using your value for $\theta$ from (b), and the formula from (a), calculate the time taken if the lake has a diameter of 100 m.

If $b = 100$ then

$$T = \frac{\sqrt{3}(100)}{2} + \frac{\pi(100)}{12}$$

$$\approx 112$$ seconds.
Section D

Answer ALL questions in this section.

This section assesses Criterion 6.

Question 33

(2 marks)

Given that \[ \int_{1}^{3} (F(x)) \, dx = 4 \], what is the exact value of \[ \int_{1}^{3} (F(x) + 2x) \, dx \]?

\[
= \int_{1}^{3} F(x) \, dx + \int_{1}^{3} 2x \, dx
\]

\[
= 4 + \left[ x^2 \right]_{1}^{3}
\]

\[
= 12
\]

Question 34

(4 marks)

The rate of population growth of a particular country is modelled by \( \frac{dP}{dt} = 0.72e^{0.03t} \), where \( t \) is the time in years since 1 January 1970 and \( P \) is the population in millions.

(a) If the population on 1 January 1970 was estimated to be 27 million, determine an expression, \( P(t) \), that models the population of this country.

\[ P(t) = \int_{0}^{t} 0.72e^{0.03t} \, dt \]

\[
= 0.72 \frac{e^{0.03t}}{0.03} + C
\]

\[ \text{at } t = 0, \quad P = 27 \]

\[
\therefore C = 3
\]

\[ \therefore P(t) = 24e^{0.03t} + 3 \]

(b) Hence, determine the predicted population of the country on 1 January 2012.

Express your answer to the nearest million.

\[ P(42) = 24e^{0.03 \times 42} + 3 \]

\[ = 87.6 \text{ million} \]

Section D continues opposite.
Section D (continued)

Question 35

Graphs of \( g(x) = 5 - |x - 2| \) and \( f(x) = (x - 2)^2 + 3 \) are shown below.

(a) **Shade** in the region with area \( \int_{a}^{b} (g(x) - f(x)) \, dx \).

(b) Determine both points of intersection.

\[
g(x) = f(x)
\]

when \( x=1 \) and \( x=3 \), \( g(3) = f(3) = 4 \) (from calculator)

\( g(1) = f(1) = 4 \) \hspace{1cm} \text{Points of intersection are (1, 4) and (3, 4)}

(c) Hence find the exact value of the area shaded.

\[
\text{Area} = \int_{1}^{3} \left( 5 - |x - 2| - (x - 2)^2 - 3 \right) \, dx
\]

\[
= \frac{7}{3} \quad \text{(from calculator)}
\]

Section D continues over the page.
Section D (continued)

Question 36

If \( h(x) = 2 \log_e(\cos x) \):

(a) find an expression for \( h'(x) \) in terms of \( \tan x \).

\[
\begin{align*}
h'(x) &= 2 \cdot \frac{1}{\cos x} (-\sin x) \\
&= -2 \cdot \tan x
\end{align*}
\]

(b) use your result from (a) to show that \( \int_{0}^{\frac{\pi}{3}} (\tan x) \, dx = \log_e 2 \).

\[
\begin{align*}
\int_{0}^{\frac{\pi}{3}} \tan x \, dx &= \frac{1}{2} \int_{0}^{\frac{\pi}{3}} -2 \tan x \, dx \\
&= \frac{1}{2} \left[ 2 \log_e(\cos x) \right]_{0}^{\frac{\pi}{3}} \\
&= \left( \log_e(\cos \frac{\pi}{3}) - \log_e(\cos 0) \right) \\
&= \left( \log_e(\frac{1}{2}) - \log_e(1) \right) \\
&= \log_e(\frac{1}{2}) \\
&= \log_e(2^{-1}) \\
&= \log_e 2
\end{align*}
\]
Section E

Answer ALL questions in this section.

This section assesses **Criterion 7**.

**Question 37**  
(2 marks)

A fair coin is tossed three times and the number of heads (H) and number of tails (T) are recorded.

(a) List the sample space.

```
HHH, HHT, HTH, HTT, TTH, THH, THT, TTH, TTT
```

(b) State the probability of getting exactly two heads.

\[
P(\text{2 heads}) = \frac{3}{8}
\]

**Question 38**  
(4 marks)

Assume that the probability of a randomly chosen person having their birthday in January is \(\frac{1}{12}\).

If \(N\) people are chosen at random, find the smallest integer value of \(N\) so the probability that at least one person has a birthday in January exceeds 0.75.

**Distribution is binomial with \(n\) trials**

**Probability distribution is given by**

\[
\left(\frac{11}{12}\right)^n = n_c \left(\frac{11}{12}\right)^n \cdot \left(\frac{1}{12}\right)^{n-c} + \ldots
\]

\[
P(\text{at least 1}) = 1 - P(0)
\]

\[
= 1 - \left(\frac{11}{12}\right)^n
\]

\[
1 - \left(\frac{11}{12}\right)^n > 0.75
\]

\[
0.25 > \left(\frac{11}{12}\right)^n
\]

\[
\log(0.25) > \log\left(\frac{11}{12}\right)^n
\]

\[
\log(0.25) < -n
\]

\[
\log\left(\frac{11}{12}\right) > \frac{1}{5}
\]

\(n > 5.9\)

Section E continues opposite.
Section E (continued)

Question 39

There are three boxes. One contains 20 items, one contains 200 items and one contains 2000 items. 10% of items are said to be defective. Sample sizes of 12 are removed from each box, both with and without replacement.

(a) Given that $X =$ number of defective items, complete the table below by determining the missing probabilities.

*You may use your calculator to provide answers to 4 decimal places without giving reasoning.*

<table>
<thead>
<tr>
<th>Sampling</th>
<th>Without Replacement</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>20</td>
<td>200</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>$\Pr(X = 0)$</td>
<td>0.1474</td>
<td>0.2718</td>
<td>0.2814</td>
<td>0.2824</td>
</tr>
<tr>
<td>$\Pr(X = 1)$</td>
<td>0.5053</td>
<td>0.3860</td>
<td>0.3775</td>
<td>0.3766</td>
</tr>
<tr>
<td>$\Pr(X \leq 1)$</td>
<td>0.6526</td>
<td>0.6579</td>
<td>0.6589</td>
<td>0.6590</td>
</tr>
</tbody>
</table>

(b) Comment upon comparisons between binomial and hypergeometric distributions as the population size increases using probabilities from the table above.

As the size of population from which the sample is drawn increases, a binomial distribution approximates a hypergeometric distribution more accurately.

Section E continues over the page.
Section E (continued)

Question 40

One of the laws of cricket states that a new match ball must not weigh less than 155.9 g or more than 163.0 g.

A leading Australian cricket ball manufacturer has found that the weights of balls they made are normally distributed and that, on average, 5% are rejected as underweight whilst 7% are rejected as overweight.

(a) Determine the standardised z scores associated with each of the percentages given above.

Express answers to 3 decimal places.

\[ P(z < Z_{1}) = 0.05 \]
\[ P(z > Z_{2}) = 0.07 \]

\[ Z_{1} = -1.645 \]
\[ Z_{2} = 1.476 \]

(b) Use your answers from (a) to write two equations in the form \( z\sigma + \mu = x \) and solve these to find the actual mean and standard deviation.

Express answers to 3 decimal places.

\[ Z_{1} = \frac{x_{1} - \mu}{\sigma} \]
\[ Z_{2} = \frac{x_{2} - \mu}{\sigma} \]

\[ -1.645 = \frac{155.9 - \mu}{\sigma} \]
\[ 1.476 = \frac{163.0 - \mu}{\sigma} \]

\[ -1.645\sigma + \mu = 155.9 \]
\[ 1.476\sigma + \mu = 163.0 \]

\[ \mu = 159.642 \]
\[ \sigma = 2.275 \]

(c) For international cricket matches only premium balls must be used and their weights must be between 159.0 g and 160.0 g.

Use your mean and standard deviation values to determine the percentage of balls suitable for international matches.

Express answer to the nearest percentage.

\[ P(159.0 < x < 160.0) = 0.1736 \]

\[ \approx 17.36\% \text{ of cricket balls from this manufacturer are suitable} \]
PLACE LABEL HERE

Possible Solutions

Tasmanian Certificate of Education
MATHEMATICS – METHODS

Senior Secondary

Subject Code: MTM315109

External Assessment

2012

Part 2
Calculators are allowed to be used

Time: 100 minutes

On the basis of your performance in this examination, the examiners will provide a result on the following criteria taken from the syllabus statement:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
<th>For Marker Use Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Demonstrate an understanding of polynomial, hyperbolic, exponential, and logarithmic functions.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Demonstrate an understanding of circular functions.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Use differential calculus in the study of functions.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Use integral calculus in the study of functions.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Demonstrate an understanding of binomial, hypergeometric and normal probability distributions.</td>
<td></td>
</tr>
</tbody>
</table>

Section A

This section assesses Criterion 3.

Question 21 (2 marks)

Given \( f(x) = 2x^2 - 3x - 4 \) and \( g(x) = 3\sqrt{x} \).

(a) Evaluate \( f[4] \).  
\[
f(4) = 2(4)^2 - 3(4) - 4 = 32
\]

(b) Evaluate \( g(4) \).
\[
g(4) = 3\sqrt{4} = 6
\]

Question 22 (4 marks)

The graph of the function \( y = f(x) \) is shown below.

On the axes to the right, sketch a graph of its inverse function, \( y = f^{-1}(x) \).

Label the asymptotes and the image of point A.

---

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Section A (continued)

Question 23
(4 marks)

In the first five years after planting, the growth of the trunk of a particular species of tree can be modelled by the equation \( D = D_010^{kr} \), where \( r \) is the time in years after planting, \( D \) is the diameter of the trunk in centimetres and \( D_0 \) is the diameter at the time of planting.

The diameter after one year is 50 cm and after three years is 70 cm.

(a) Calculate the exact value of the constant \( k \).

\[ t=1 \quad \text{gives} \quad 50 = D_0 \times 10^k \]  \( \text{(a)} \) \( \rightarrow \frac{5}{2} \)

\( t=3 \quad \text{gives} \quad 70 = \frac{D_0}{100} \times 10^{3k} \)  \( \text{(b)} \) \( \rightarrow \frac{3}{2} \)

\( \text{(b)/(a)} \text{ gives } \frac{70}{50} = \frac{D_0}{100} \times 10^{3k} \times 10^k \)

\[ k = \log \left( \frac{7}{5} \right) \text{ working not essential} \]

\[ k = \frac{\log (\frac{7}{5})}{2} \rightarrow (1) \]

(b) Hence show that \( D_0 \) equals \( \frac{\sqrt{50}}{7} \).

At \( t=1 \) \[ 50 = D_0 \times 10^{\frac{5}{2}} \]

\[ D_0 = \frac{50}{10^{\frac{5}{2}}} \]

\[ D_0 = \frac{50}{(\frac{5}{2})^5} \rightarrow \frac{1}{2} \text{ working not essential} \]

\[ D_0 = \frac{50}{\sqrt{5}} \]

\[ D_0 = \frac{50}{5} \]

\[ D_0 = \frac{50}{5} = 10 \]

\[ \sqrt{50} = \sqrt{5^2} \times \sqrt{2} = 5\sqrt{2} \]

Section A continues over the page.

Section A (continued)

Question 24
(6 marks)

A function is of the form \( f(x) = \frac{a}{(x-h)^2} + k \).

(a) Given that \( x = 3 \) and \( y = 5 \) are asymptotes and that \( (2, 3) \) is a point on the graph of the function, determine values for the constants \( a, h, \) and \( k \).

\[ VA \ (x=3) \Rightarrow h=3 \] \( (1) \)

\[ HA \ (y=5) \Rightarrow k=5 \] \( (1) \)

Solve \[ \frac{a}{(x-3)^2} + 5 \] given \( (2, 3) \) point.

\[ \frac{a}{(2-3)^2} + 5 \]

\[ 2 = 5 \]

\[ a = -2 \] \( (1) \)

(b) Hence determine fully simplified, exact value solutions of the equation \( f(2x+1) = \frac{9}{2} \).

\[ f(2x+1) = \frac{9}{(2x+1-3)^2} + 5 \]

\[ f(2x+1) = -\frac{2}{(2x-2)^2} + 5 \]

\[ \text{Solve } (2x-2)^2 + 5 = \frac{4}{5} \rightarrow (1) \]

\[ (2x-2)^2 = -\frac{1}{5} \]

\[ 2x-2 = \pm \frac{\sqrt{5}}{5} \]

\[ 2x-2 = \pm \frac{\sqrt{5}}{5} \]

\[ 2x = 2 \pm \frac{\sqrt{5}}{5} \]

\[ x = 1 \pm \frac{\sqrt{5}}{10} \]

\[ x = 2 \] \( \text{or} \) \[ x = 0 \] \( (1) \)
Section B

Answer ALL questions in this section.

This section assesses Criterion 4.

Question 25
(a) State the number of solutions of the equation \( \sin(2\pi x) = \frac{1}{2} \) in \([-1, 1]\).

4 solutions

(b) Write down one of these solutions.

\( x = -0.9598 \) or \( x = 0.5402 \) or \( x = 0.9402 \) or \( x = 0.4598 \)

Question 26
The diagram below shows the first quadrant of a unit circle with \( \angle COD = \theta \).

OCD and OAB are similar right angled triangles and \( AB = 2.246 \) units

(a) Determine \( \theta \) to 3 decimal places.

\[ \tan \theta = \frac{2.246}{1} \]

\[ \theta = \tan^{-1}(2.246) = 1.152 \] (i)

(b) Determine to 3 decimal places:

(i) \( \overrightarrow{OD} = (\cos(1.152), 0.407) \)

(ii) \( \overrightarrow{DC} = (\sin(1.152), 0.914) \)

(c) Use the answers from (b) to show \( \sin^2 \theta + \cos^2 \theta = 1 \).

Remember: Using answers to 3 decimal places will not produce the exact value.

\[ \sin^2 \theta + \cos^2 \theta = (\sin \theta)^2 + (\cos \theta)^2 \]

\[ = (0.914)^2 + (0.407)^2 \]

\[ = 1.001 \approx 1 \] (i)

Section B continues opposite.
Section B (continued)

Question 28

(a) Express the equation \( \sqrt{3} \cos \left(2x - \frac{\pi}{2}\right) = \sin \left(2x - \frac{\pi}{2}\right) \) in the form \( \tan(2x - b) = c \).

\[
\begin{align*}
\sin \left(2x - \frac{\pi}{2}\right) &= \sqrt{3} \\
\cos \left(2x - \frac{\pi}{2}\right) &= 0 \\
\therefore \tan \left(2x - \frac{\pi}{2}\right) &= \sqrt{3} & \text{NO PART MARKS}
\end{align*}
\]

(b) Hence, determine the exact value solutions of \( \sqrt{3} \cos \left(2x - \frac{\pi}{2}\right) = \sin \left(2x - \frac{\pi}{2}\right) \) over the interval \([0, \pi]\).

- **Basic angle** = \( \frac{\pi}{3} \) \( (1) \) 
- **Allocated part marks**
- **Solve** \( 2x - \frac{\pi}{2} = \frac{\pi}{3} \) 
- **Explicit different solutions styles**
- \( 2x = \frac{5\pi}{6} \)
- \( x = \frac{5\pi}{12} \) \( (1) \) 

- **Answer only**
- **Solve** \( 2x - \frac{\pi}{2} = \pi + \frac{\pi}{3} \) \( (1) \)
- \( 2x = \frac{4\pi}{3} + \frac{\pi}{2} \)
- \( 2x = \frac{11\pi}{6} \)
- \( x = \frac{11\pi}{12} \) \( (1) \)

- **Over** \([0, \pi]\) \( x = \frac{5\pi}{12} \) OR \( x = \frac{11\pi}{12} \)

---

Question 28 (continued)

(c) Using solutions from (b), determine the exact values of the coordinates of the points of intersection of \( f(x) = \sqrt{3} \cos \left(2x - \frac{\pi}{2}\right) \) and \( g(x) = \sin \left(2x - \frac{\pi}{2}\right) \) over the domain \([0, \pi]\).

- **Solutions from (b) represent** \( x \) values of intersection points.
- Substitute into either \( f(x) \) or \( g(x) \) to find corresponding \( y \) values.
- If \( x = \frac{5\pi}{12} \) \( g \left( \frac{5\pi}{12} \right) = \sin \left( \frac{4\pi}{12} - \frac{\pi}{3} \right) = \sin \left( \frac{\pi}{3} \right) = \frac{\sqrt{3}}{2} \)
- \( \left( \frac{5\pi}{12}, \frac{\sqrt{3}}{2} \right) \) point of intersection
- Similarly substituting \( \frac{11\pi}{12} \) gives
- \( \left( \frac{11\pi}{12}, -\frac{\sqrt{3}}{2} \right) \) point of intersection

\[ \text{Answer only} = 1. \]
- **Jot/Scratch required**
- **Approximate values** = 1.
Answer ALL questions in this section.

This section assesses Criterion 5.

**Question 29**

(2 marks)

Find the $x$-coordinates of the points on the curve $y = x^3 - 2x^2 + 5$ where the tangents to the curve are parallel to the line $y = 7 - x$.

\[ \frac{dy}{dx} = 3x^2 - 4x \quad \text{and} \quad y = 7 - x \quad \text{has gradient} \quad -1 \]

Solve $3x^2 - 4x - 1 = 0$

\[ \begin{align*}
3x^2 - 4x + 1 &= 0 \\
(3x - 1)(x - 1) &= 0 \\
\therefore x &= \frac{1}{3} \quad \text{and} \quad x = 1
\end{align*} \]

**Question 30**

(4 marks)

A TV satellite company has 10 000 customers each paying $25 per month. Research has shown the company is likely to gain 500 more customers for every $1 the monthly fee drops.

(a) Show that the revenue per month is $R = -500x^2 + 2500x + 250000$, where $x$ represents the drop in the monthly fee in dollars.

\[ R = (\text{customers}) \times (\text{monthly fee}) \]

\[ R = (10000 + 500x) \times (25 - x) \]

\[ R = -500x^2 + 10000x + 250000 - 12500x + 250000x \]

\[ R = -500x^2 + 2500x + 250000 \]

(b) Apply differential calculus techniques to determine the fee per month that will provide maximum revenue. There is no need to justify that it is a maximum.

\[ R' = -1000x + 2500 \]

Maximum when $R'(x) = 0$

Solve $-1000x + 2500 = 0$

\[ x = 2.5 \]

Maximum revenue when fee is $25 - 2.5 = $22.50

Section C continues opposite.
Section C (continued)

Question 31

Use the axes provided on the right to sketch the gradient functions for both \( y = f(x) \) and \( y = g(x) \) shown below.

Highlight any discontinuities by clearly marking points or asymptotes on the sketches.

(a)

(b)
Section C (continued)

Question 32  

Veronica Smith is at point A on the edge of a circular lake of diameter \(b\) metres.

She swims to B at 1 m/s and then walks at 2 m/s along the lake’s edge to C where she meets Tabitha Jones.

\(ABC\) is a right-angled triangle, angle \(BAC = \theta\) and angle \(BOC = 2\theta\). The length of the arc \(BC\) equals the length of the radius multiplied by the angle subtended at the centre by the arc.

\(\begin{align*}
T(\theta) &= b \cos \theta + \frac{b \theta}{2} \\
\text{SWIM:} & \quad \cos \theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{AB}{b} \implies AB = b \cos \theta \\
T_{\text{swim}} &= \frac{\text{DIST.}}{\text{SPEED}} = \frac{AB}{1} = b \cos \theta \\
\text{WALK DISTANCE} &= BC = \text{ARC} = \text{radius} \times \text{angle (centre)} \\
&= \frac{b}{\theta} \times 2\theta = b \theta \\
T_{\text{walk}} &= \frac{\text{DIST.}}{\text{SPEED}} = \frac{BC}{2} = \frac{b \theta}{2} \\
T(\theta) &= T_{\text{swim}} + T_{\text{walk}} = b \cos \theta + \frac{b \theta}{2}
\end{align*}\)

Question 32 continues opposite.
Question 32 (continued)

(b) Determine an expression for \( T'(\theta) \) and use this derivative to calculate an exact value for \( \theta \) where a stationary point exists. State the nature of this stationary point and justify your answer.

\[ T'(\theta) = -6\sin \theta + \frac{b}{3} \]

\[ \frac{1}{3} + \frac{1}{2} = 1 \]

\[ \text{S.P when } T'(\theta) = 0 \]

\[ -6\sin \theta + \frac{b}{3} = 0 \]

\[ -6\sin \theta = \frac{-b}{3} \]

\[ \sin \theta = \frac{b}{6} \]

\[ \theta = \frac{\pi}{6} \]

\[ \text{Justify maximum using gradient table where } \]

\[ T'(\theta) = 6(\sin \theta + \frac{1}{2}) \text{ so as } b > 0 \text{ consider the change in sign of } (\sin \theta + \frac{1}{2}) \text{ passing through } \theta = \frac{\pi}{6} \]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0</th>
<th>( \frac{\pi}{6} )</th>
<th>( \frac{\pi}{6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \theta + \frac{1}{2})</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Slope</td>
<td>1</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Maximum at \( \theta = \frac{\pi}{6} \)

(c) Using your value for \( \theta \) from (b), and the formula from (a), calculate the time taken if the lake has a diameter of 100 m.

\[ \beta = 100 \]

\[ \therefore T(\beta) = 100 \cos(\frac{\pi}{6}) + \frac{100 \times \frac{\pi}{6}}{2} \]

\[ = 100 \times \frac{\sqrt{3}}{2} + \frac{50\pi}{6} \]

\[ = 50\sqrt{3} + 25\pi \approx 117.78 \]

\[ \approx 113 \text{ seconds} \]

Page 15
Answer ALL questions in this section.

This section assesses Criterion 6.

Question 33 (2 marks)

Given that \( \int_{1}^{3} (F(x) + 2x) \, dx = 4 \), what is the exact value of \( \int_{1}^{3} (F(x) + 2x) \, dx \)?

\[
\int_{1}^{3} (F(x) + 2x) \, dx = \int_{1}^{3} F(x) \, dx + \int_{1}^{3} 2x \, dx
\]

\[
= \left[ F(x) \right]_{1}^{3} + \left[ x^2 \right]_{1}^{3} \frac{1}{2}
\]

\[
= 4 + \left[ 3^2 - 1^2 \right] \frac{1}{2} = 12
\]

Question 34 (4 marks)

The rate of population growth of a particular country is modelled by \( \frac{dp}{dt} = 0.72e^{0.03t} \), where \( t \) is the time in years since 1 January 1970 and \( P \) is the population in millions.

(a) If the population on 1 January 1970 was estimated to be 27 million, determine an expression, \( P(t) \), that models the population of this country.

\[
P = \int \frac{dp}{dt} \, dt = \int 0.72e^{0.03t} \, dt = \frac{0.72e^{0.03t}}{0.03} + C
\]

\[
\therefore P = 24e^{0.03t} + C \quad (e^{0.03} \approx 1.03045)
\]

\[
t = 0 \quad P = 27
\]

\[
27 = 24 + C + C \quad \therefore C = 3
\]

\[
\therefore P = 24e^{0.03t} + 3
\]

(b) Hence, determine the predicted population of the country on 1 January 2012.

Express your answer to the nearest million.

\[
\text{1 January 2012} \Rightarrow t = 42
\]

\[
P(42) = 24e^{0.03(42)} + 3 \approx 87.6
\]

\[
\text{Pop. 1 Jan, 2012} \approx 88 \text{ million}
\]
Section D (continued)

Question 35

(4 marks)

Graphs of \( g(x) = 5 - |x - 2| \) and \( f(x) = (x - 2)^2 + 3 \) are shown below.

(a) **Shade in the region with area** \( \int_{a}^{b} (g(x) - f(x)) \, dx \).

(b) Determine both points of intersection.

\[
\text{Solve} \quad 5 - |x - 2| = (x - 2)^2 + 3
\]

\[
\text{gives} \quad x = 1 \text{ and } x = 3
\]

\( \therefore \) \( x = 1 \) and \( x = 3 \) are equivalent.

\( \therefore (1, 4) \text{ and } (3, 4) \) points of intersection

(c) Hence find the exact value of the area shaded.

\[
\text{Shaded area} = \int_{1}^{3} \left[ (5 - |x - 2|) - ((x - 2)^2 + 3) \right] \, dx
\]

\[
= \int_{1}^{3} \left[ 2 \right] \, dx = \left[ 2x \right]_{1}^{3} = 4 - 2 = 2 \text{ units}^2
\]

Section D continues over the page.
Section D (continued)

Question 36

If \( h(x) = 2 \log_e (\cos x) \):

(a) find an expression for \( h'(x) \) in terms of \( \tan x \).

\[
\frac{h'(x)}{\cos x} = 2 \frac{\sin x}{\cos x} \quad \text{or} \quad h'(x) = -2 \tan x
\]

(b) use your result from (a) to show that \( \int_0^{\frac{\pi}{3}} (\tan x)dx = \log_e 2 \).

\[
\int h'(x) \, dx = h(x) + C
\]

\[
\int -2 \tan x \, dx = 2 \log_e (\cos x) + C
\]

\[
\frac{\pi}{3} \tan x \, dx = - \left[ \log_e (\cos x) \right]_{\frac{\pi}{3}}^0
\]

\[
= - \left( \log_e (\cos \frac{\pi}{3}) - \log_e (\cos 0) \right) = - \left( \log_e \frac{1}{2} - \log_e 1 \right) = - \log_e \frac{1}{2} = \log_e 2^{-1} = \log_e 2
\]
Answer ALL questions in this section.

This section assesses Criterion 7.

Question 37

A fair coin is tossed three times and the number of heads (H) and number of tails (T) are recorded.

(a) List the sample space.
\[
\{TTT, TTH, THH, HHT, HTH, THH, HTT, TTT\}
\]

(b) State the probability of getting exactly two heads.
\[X = \text{HHT, HTT, THT, THH} \quad P(X=2) = \frac{3}{8} = 0.375\]

Question 38

Assume that the probability of a randomly chosen person having their birthday in January is \(\frac{1}{12}\).

If \(N\) people are chosen at random, find the smallest integer value of \(N\) so that the probability that at least one person has a birthday in January exceeds 0.75.

\[X = \text{Jan. Birthdays} \quad P = \frac{1}{12} \quad \text{"at least one"} = 1 - P(X=0)\]

\[
\begin{align*}
\text{Solve} & \quad 1 - P(X=0) > 0.75 \\
\Rightarrow & \quad 1 - \left(\frac{11}{12}\right)^N > 0.75 \\
\Rightarrow & \quad \left(\frac{11}{12}\right)^N > 0.25 \\
\text{Solving} & \quad \left(\frac{11}{12}\right)^N = 0.75 \quad \text{gives} \quad N \approx 15.93 \\
1^\circ & \quad N = 16 \Rightarrow 1 - \left(\frac{11}{12}\right)^{16} = 0.75 \\
2^\circ & \quad N = 15 \Rightarrow 1 - \left(\frac{11}{12}\right)^{15} = 0.729 \\
\Rightarrow & \quad 16 \text{ is smallest integer value} \\
\end{align*}
\]
Section E (continued)

Question 39

There are three boxes. One contains 20 items, one contains 200 items and one contains 2000 items. 10% of items are said to be defective. Sample sizes of 12 are removed from each box, both with and without replacement.

(a) Given that \( X \) = number of defective items, complete the table below by determining the missing probabilities.

*You may use your calculator to provide answers to 4 decimal places without giving reasoning.*

<table>
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<tr>
<th>Sampling Items</th>
<th>Without Replacement</th>
<th>With Replacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr(( X = 0 ))</td>
<td>0.1474</td>
<td>0.2718</td>
</tr>
<tr>
<td>Pr(( X = 1 ))</td>
<td>0.5053</td>
<td>0.3860</td>
</tr>
<tr>
<td>Pr(( X \leq 1 ))</td>
<td>0.6527</td>
<td>0.6578</td>
</tr>
</tbody>
</table>

(b) Comment upon comparisons between binomial and hypergeometric distributions as the population size increases using probabilities from the table above.

- Hypergeometric (without replacement) Binomial (with replacement)
- As \( N \) gets large in reference to \( n \) then binomial approximation for hypergeometric becomes more valid
- \( \text{Pr}(X=0) \) for \( N=20 \) 0.1474 nearly half of 0.2814
- Clearly probabilities for \( N=200 \) are better
  - with \( N=2000 \) for \( X=0, X=1, X=1 \) hypergeometric values quite close to corresponding binomial values

Section E continues over the page.
Section E (continued)

Question 40

One of the laws of cricket states that a new match ball must not weigh less than 155.9 g or more than 163.0 g.

A leading Australian cricket ball manufacturer has found that the weights of balls they made are normally distributed and that, on average, 5% are rejected as underweight whilst 7% are rejected as overweight.

(a) Determine the standardised $z$ scores associated with each of the percentages given above.

Express answers to 3 decimal places.

\[
\begin{align*}
Z_{u} &= -1.645 \quad \text{inv NORMCDF}(1.0, 0.05, 0.0) \\
Z_{o} &= 1.476 \quad \text{inv NORMCDF}(0.9, 0.07, 1.0)
\end{align*}
\]

(b) Use your answers from (a) to write two equations in the form $z\sigma + \mu = x$ and solve these to find the actual mean and standard deviation.

Express answers to 3 decimal places.

\[
\begin{align*}
\frac{1}{2} \quad -1.645 \sigma + \mu &= 155.9 \\
\frac{1}{2} \quad 1.476 \sigma + \mu &= 163.0
\end{align*}
\]

Solving simultaneous equations gives

\[
\begin{align*}
\mu &= 159.642 \quad \text{g} \\
\sigma &= 2.275 \quad \text{g}
\end{align*}
\]

(c) For international cricket matches only premium balls must be used and their weights must be between 159.0 g and 160.0 g.

Use your mean and standard deviation values to determine the percentage of balls suitable for international matches.

Express answer to the nearest percentage.

\[
\begin{align*}
\frac{1}{2} \quad P(159.0 < x < 160.0) &= 0.1736 \\
\frac{1}{2} \quad \text{Approximately 17% of cricket balls deemed suitable for international matches}
\end{align*}
\]
TASMANIAN QUALIFICATIONS AUTHORITY

MTM315109 Mathematics Methods

ASSESSMENT PANEL REPORT

Award Distribution

<table>
<thead>
<tr>
<th></th>
<th>EA</th>
<th>HA</th>
<th>CA</th>
<th>SA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>This year</td>
<td>13%</td>
<td>18%</td>
<td>38%</td>
<td>32%</td>
<td>591</td>
</tr>
<tr>
<td>Last year</td>
<td>15%</td>
<td>17%</td>
<td>37%</td>
<td>31%</td>
<td>601</td>
</tr>
<tr>
<td>Last year (all examined subjects)</td>
<td>11%</td>
<td>19%</td>
<td>39%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>Previous 5 years</td>
<td>15%</td>
<td>18%</td>
<td>36%</td>
<td>32%</td>
<td></td>
</tr>
<tr>
<td>Previous 5 years (all examined subjects)</td>
<td>11%</td>
<td>19%</td>
<td>40%</td>
<td>30%</td>
<td></td>
</tr>
</tbody>
</table>

Student Distribution (SA or better)

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Year 11</th>
<th>Year 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>This year</td>
<td>60% (352)</td>
<td>40% (239)</td>
<td>73% (431)</td>
<td>27% (160)</td>
</tr>
<tr>
<td>Last year</td>
<td>56% (335)</td>
<td>44% (266)</td>
<td>72% (434)</td>
<td>28% (165)</td>
</tr>
<tr>
<td>Previous 5 years</td>
<td>58%</td>
<td>42%</td>
<td>63%</td>
<td>37%</td>
</tr>
</tbody>
</table>