Candidate Instructions

1. You MUST make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.

2. Answer ALL questions. Answers must be written in the spaces provided on the examination paper.

3. You should make sure you answer all parts within each question so that the criterion can be assessed.

4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.

5. The 2016 External Examination Information Sheet for Mathematics Methods can be used throughout the examination. No other written material is allowed into the examination.

6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

- **Criterion 4**: Demonstrate an understanding of polynomial, hyperbolic, exponential and logarithmic functions.
- **Criterion 5**: Demonstrate an understanding of circular functions.
- **Criterion 6**: Use differential calculus in the study of functions.
- **Criterion 7**: Use integral calculus in the study of functions.
- **Criterion 8**: Demonstrate an understanding of binomial, hypergeometric and normal probability distributions.
This part of the examination is worth 60 marks in total. Each section is worth 12 marks.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 or 2 marks, whilst no workings are required, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 3 or more marks, you are **required to show** relevant working.
Answer ALL questions in this section.

This section assesses Criteria 4.

Question 1
State a possible graph for the following function (in factorised form): (2 marks)

Question 2
State the domain and range of the circle with equation given by: (2 marks)

\[(x - 5)^2 + (y + 2)^2 = 16\]

Domain: 
Range: 

Section A continues.
Section A (continued)

Question 3

The square root graph on the left is transformed into the square root graph on the right using two reflections followed by two translations.

(a) State the two reflections required.  

(b) State the two translations required.  

(c) The square root graph on the left is given by $f(x) = 2\sqrt{x} + 4 + 1$. Find the equation of the square root graph on the right.  

Section A continues.
Section A (continued)

Question 4

Two functions $f$ and $g$ are given by $f(x) = \log_3 \left( \frac{x^3}{3} \right)$ and $g(x) = \sqrt[3]{3^{x+1}}$.

(a) Show that the composite function, $f \circ g$, is equal to $x$, i.e. $(f \circ g)(x) = x$.

(3 marks)

(b) Given that $g(f(x)) = x$ and your result in part (a), what does this say about the functions $f$ and $g$?

(1 mark)
Answer **ALL** questions in this section.

This section assesses **Criteria 5**.

**Question 5**

(a) Evaluate $\sin\left(\frac{3\pi}{2}\right) \times \cos(0) - \tan(2\pi) \times \sin\left(\frac{\pi}{2}\right)$. (2 marks)

(b) Evaluate $\cos(135^\circ) \times \sin(330^\circ)$. (2 marks)

**Question 6**

Use the Multiple Angle Formulae to simplify $\sin\left(\frac{x - \pi}{2}\right)$ as far as possible. (2 marks)

Section B continues.
Section B (continued)

Question 7

Find an equation that represents the graph below. Express your answer in the form $y = a \sin(bx) + c$ where $a$, $b$ and $c$ are real numbers. (3 marks)

![Graph of a sine function]

Question 8

Solve the equation $\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ for $x \in [0, 2\pi]$. (3 marks)
Answer ALL questions in this section.

This section assesses Criteria 6.

Question 9

Find the derivative of \( f(x) = 6x^3 + 5x + 3\sqrt{x} \). (2 marks)

Question 10

A student prepares the following table for a ‘change of sign of derivative test’:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>f'(x)</td>
<td>-128</td>
<td>0</td>
<td>24</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>-200</td>
</tr>
</tbody>
</table>

Give the \( x \) coordinate of the stationary points of \( f(x) \) and state their nature. (3 marks)
Section C (continued)

Question 11

Find the derivative of \( y = 6 \sin^2 (5x) \). (2 marks)

Question 12

Using the change of base rule for logarithms, \( \log_a B = \frac{\log_x B}{\log_x a} \), find the derivative of \( y = \log_2 (3x) \). (2 marks)

Question 13

Find the tangent to the equation \( f(x) = \frac{x^2}{e^x} \) at the point \( \left( 1, \frac{1}{e} \right) \). (3 marks)
Answer ALL questions in this section.

This section assesses **Criteria 7**.

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**Question 14**

Find \( \int \frac{(3-x)^2}{5} \, dx \)  

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**Question 15**

The two functions \( f(x) = x^2 + 3x + 4 \) and \( g(x) = x^2 + 3x - 2 \) are drawn below for \( x \in [-1, 3] \).

Find the area between the two curves between \( x = -1 \) and \( x = 3 \).
Section D (continued)

Question 16
Determine the value of \( \int_{\pi/6}^{\pi/4} \frac{\sin(3x)}{2} \, dx \). (2 marks)

Question 17
Differentiate \( y = x \ln x \) and use this result to find \( \int \ln x \, dx \). (3 marks)

Question 18
Find \( k \) such that \( \int_{1}^{k} \frac{3}{2x - 1} \, dx = \frac{3 \ln 5}{2} \), for \( k > 1 \). (3 marks)
Answer **ALL** questions in this section.

This section assesses **Criteria 8**.

**Question 19**

The length of leaves on a tree is found to fit a normal distribution $X$ with mean 15 cm.

Given that the probability of a leaf being 17 cm or longer is 0.3 ([ie. $Pr(X \geq 17) = 0.3$](#))

(3 marks)

Calculate:

(a) $Pr(X \leq 17)$

(b) $Pr(X \leq 13)$

(c) $Pr(13 \leq X \leq 15)$

Section E continues.
Section E (continued)

Question 20

A spinner with sectors marked 1 to 4 is spun twice and the greater of the two results recorded.

For example if 2 and 3 are spun, 3 is recorded, if 4 and 4 are spun, 4 is recorded.

(a) Complete the following table of possible outcomes. (1 mark)

<table>
<thead>
<tr>
<th>First Spin</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

(b) Complete the probability distribution table for the distribution X. Leave your answers as fractions. (1 mark)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td></td>
<td></td>
<td>( \frac{5}{16} )</td>
<td></td>
</tr>
</tbody>
</table>

(c) Calculate the Expected Value, E(X), of the distribution in part (b). (2 marks)

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Question 20 continues.
Question 20 (continued)

(d) A person uses the spinner to obtain 10 results. What is an expression for the probability that 4 of the results are 3?

Leave your answer in the form \( C \cdot (p)^z \cdot (1-p)^{n-z} \). (2 marks)

Question 21

The Senate of a country consists of 45 men and 65 women. (3 marks)

5 Senators are to be selected at random to form a committee.

If \( X \) is the random variable for the number of men in the committee, what is \( \Pr(X \geq 1) \), i.e. the probability that the committee includes at least one man?

Note that \( C \), terms can be left not simplified in your answer.
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MATHEMATICS METHODS
(MTM315114)

PART 2

Calculators are allowed to be used

Time: 100 minutes

Candidate Instructions

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2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.

3. You should make sure you answer all parts within each question so that the criterion can be assessed.

4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.

5. The 2016 External Examination Information Sheet for Mathematics Methods can be used throughout the examination. No other written material is allowed into the examination.

6. A TASC approved calculator can be used throughout this part of the examination.

7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

- **Criterion 4** Demonstrate an understanding of polynomial, hyperbolic, exponential and logarithmic functions.
- **Criterion 5** Demonstrate an understanding of circular functions.
- **Criterion 6** Use differential calculus in the study of functions.
- **Criterion 7** Use integral calculus in the study of functions.
- **Criterion 8** Demonstrate an understanding of binomial, hypergeometric and normal probability distributions.

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Additional Instructions for Candidates

This part of the examination is worth 80 marks in total. Each section is worth 16 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 or 2 marks, whilst no workings are required, markers will look at the presentation of answer(s) and at the argument(s) leading to the answer(s).

For questions worth 3 or more marks, you are **required to show** relevant working.
Answer ALL questions in this section.

This section assesses Criteria 4.

Question 22

(a) State the domain and range of the truncus pictured above. (1 mark)

Domain: .................................................................

Range: .................................................................

(b) Find an equation that represents the truncus pictured above. (3 marks)

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Section A continues.
Section A (continued)

Question 23

(6 marks)

Consider the function \( f(x) = 3e^{2x} - 1 \).

(a) Sketch the graph of the function on the axes below. Include the equation for the asymptote as well as exact values for the \( x \) and \( y \) intercepts in your sketch.

(b) Using your sketch above, determine the **number** of solutions of the equation

\[ 3e^{2x} + x^2 = 0. \]
A teenager is standing with a toy that fires foam bullets at one end of a long hallway and hits the target at the other end.

He is firing from a height of 1m and the target is at a height of 2m. The ceiling in the hall is at a height of 3m and so the maximum height of the bullet will be 3m.

Find an equation for the path of the foam bullet in the form \( y = a(x - h)^2 + k \).

Give exact values for \( a \), \( h \), and \( k \) in your answer. (6 marks)
Answer ALL questions in this section.

This section assesses Criteria 5.

Question 25

Give an equation for $y$ in terms of $x$ in the form $y = a \tan(n(x + b)) + c$, for $a, b, c, n$ real numbers. (4 marks)

Section B continues.
Section B (continued)

Question 26

Given the equation $\sqrt{3} \sin \left(2x + \frac{\pi}{4}\right) = \cos \left(2x + \frac{\pi}{4}\right)$.

(a) Write this equation in the form $\tan (ax + b) = c$.  

(b) Find all solutions of this equation for $x \in [\pi, 3\pi]$.  

(1 mark) 

(2 marks)
Section B (continued)

Question 27

The Melbourne Star is a giant ferris wheel that has a diameter of 120m, and at its lowest point is 1m above the ground.

The ferris wheel takes 30 minutes to make one revolution without stopping.

(a) A tourist enters a carriage at the bottom of the wheel at 12pm and stays on the wheel for two revolutions. At what times will they be at the top of the wheel? (1 mark)

(b) Using the grid below, sketch a graph of height of the carriage above the ground, \( H \), against time, \( t \), for the 60 minutes from 12pm. Clearly label the maximum and minimum points on your graph. (2 marks)

Question 27 continues.
Question 27 (continued)

(c) Write a suitable equation that models the relationship between $H$ and $t$ for two revolutions. 

(d) When the carriage is 85m or more above the ground they are able to see Luna Park in St Kilda. Calculate, to the nearest minute, the times they are able to see Luna Park. 

(3 marks)
Answer ALL questions in this section.

This section assesses Criteria 6.

Question 28
Find and classify any stationary points for \( f(x) = x \ln(x) \), where \( x > 0 \). (4 marks)

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Section C continues.
Section C (continued)

Question 29

A chocolate manufacturer wishes to produce a chocolate bar called the 'Chocolate Stick'. The bar is to contain a volume of 100cm³ of chocolate.

Its length is to be \( \ell = 20 \) cm.

The amount of wrapping to be used needs to be as little as possible so the surface area of the bar must be minimised.

(a) Show that the surface area is given by \( S.A = 40w + 10 + \frac{200}{w} \), where \( w \) is the width of the bar. (2 marks)

(b) Hence find the minimum surface area of the chocolate bar. (4 marks)
A new stock is to be listed for purchase and trade on the stock exchange. An economist predicts that the share price will follow the function \( P(t) = 3e^{-\frac{(t-10)^2}{100}} \), where \( P \) is the price in dollars and \( t \) is the time in weeks.

(a) Find the initial value of the stock. (1 mark)

(b) Find the rate of change of the stock after 5 weeks.

You may use your calculator to assist in finding the derivative here. (2 marks)

(c) Show using calculus that the stock will reach a maximum price after 10 weeks. (3 marks)
Answer ALL questions in this section.

This section assesses Criteria 7.

**Question 31**

(a) Sketch the graph of the two equations \( f(x) = x^3 - 3x^2 - 4x + 9 \) and \( g(x) = 2x + 1 \) including the intersections points. (2 marks)

![Graph of two curves](image)

(b) Find the area enclosed between the two curves \( f(x) \) and \( g(x) \). (3 marks)

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Section D continues.
Section D (continued)

Question 32

A shaded region is constructed from the three parabolas as shown below:

\[ f(x) = -\frac{1}{3}(x+3)(x-1), \]
\[ g(x) = \frac{1}{10}(x+1)(x-8), \text{ and} \]
\[ h(x) = -(x-7)(x-9). \]

Find the area enclosed between the three parabolas and the \( x \)-axis.

Note: You may use your calculator, but an expression involving the integrals concerned must be included in your answer. (4 marks)
Section D (continued)

Question 33

A particle has a velocity function given by \( v = 6 \cos(3t) + ke^{-2t} \), where \( k \) is a constant and \( t \) is the time in seconds.

It is known that the particle is at rest at 0 seconds.

(a) Find the value of \( k \).  (1 mark)

(b) Find an expression for the displacement for the particle, \( s \), in metres in terms of \( t \), given that the initial displacement of the particle is 2 metres.

Note that \( v = \frac{ds}{dt} \).  (3 marks)

(c) Find the total distance travelled by the particle in the first second.

Give your answer to 3 decimal places.  (3 marks)
Answer ALL questions in this section.

This section assesses Criteria 8.

Question 34

The length of nails produced by a machine are normally distributed with a mean of 100mm and a standard deviation of 1.2mm.

Determine the values $x_1$ and $x_2$, centred around the mean, such that the makers of the nails can be 90% confident that a given nail lies between these values,

ie. $\Pr(x_1 \leq X \leq x_2) = 0.90$ (3 marks)

Question 35

A bag contains 45 marbles, 10 red, 15 green and 20 blue. 6 are to be drawn out at random without replacement.

Calculate the probability of drawing exactly 2 red marbles. (2 marks)
A commuter has to drive through 8 sets of traffic lights on their way to work. They would like to model the number of red traffic lights they have to stop at with a binomial distribution.

(6 marks)

(a) What are two of the assumptions that need to be made before the binomial probability distribution can be used in this situation?

(b) The commuter is satisfied they can use the binomial distribution. If the probability of each traffic light being red is 0.4, what is the probability of at least 6 traffic lights being red on the way to work?

(c) The commuter knows that if at least 6 traffic lights are red on a given day they will be late for work.

What is the probability they will be late for work more than once in a single 5 day working week?

Section E continues.
Section E (continued)

Question 37

A camp is being held for pole vaulters in the lead up to the Olympics. The top 5%, who can jump a height of 5.80m or more, will receive automatic selection. The bottom 10%, who cannot jump above a height of 5.20m, will be sent home.

Assuming the heights which the pole vaulters can jump is normally distributed, find the mean and standard deviation of this distribution. (5 marks)
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