PART 1 – No calculators allowed

A common theme for this paper was students not reading questions correctly and therefore not answering fully or leaving out units in context-based questions. Also, attention needs to be paid to the mark allocation and ensuring there is sufficient working to gain these marks. As in previous years, student weaknesses in the basics of equation solving and fractions have influenced results.

Section A

Question 1

Most students received some marks for this question by setting up the two brackets, but some did not identify the double zero and few recognised that “a” must be negative.

Question 2

This question was reasonably well done. Students are encouraged to show working, as those who stated centre and radius and made an error with either of them were given some credit if their domain and range were a consequential error. Care must be taken using square brackets or ≤ to show that end values are included.

Question 3

a) Very well done.

b) Quite well done. Care must be taken in reading the question, as some students incorrectly stated the translations required from the graph’s original position.

c) Most students gave an equation with two or three correct elements. Many struggled to correctly reflect and translate the original function without realising that it may have been easier to find the equation from the second graph. It was pleasing to see some students check their equation using the point (2, -4).

Question 4

a) This was the most challenging question in this section. While many students correctly substituted \(g(x)\) into \(f(x)\), only those showing competence in using log laws went on to correctly simplify their equation. Care must be taken with writing and transposing indices as it was disappointing to see \(3^{x+1}\) becoming \(3^x + 1\) on the next line of working. Some students appeared unfamiliar with the \(f \circ g\) notation and tried a variety of manipulations of the two functions without success.

b) Few students recognised that the functions are inverses of each other. Some students ignored the first half of the question and made statements about the domain and range of the functions which received no mark.

Section B

Question 5

a) Very well done. Most mistakes centred around incorrect handling of the negatives. Recommend students adopt brackets involving multiplication as seems less chance to make these mistakes.

\((-1)(1)-(0)(1)\) instead of \(-1 \times 1 - 0 \times 1\)

b) Generally well done. Number of students became bogged down in either degree to radian conversions or the use of multiple angle formulae. Again, issues with the multiplication of negatives. Common mistake was
\[-\frac{\sqrt{2}}{2} x \cdot \frac{1}{2} = -\frac{\sqrt{2} - 1}{2}\]. Students struggled to find basic angle equivalents. Those who showed correct working but couldn’t simplify multiplication above were only given part mark penalty.

**Question 6**
Generally handled well. Required to use multiple angle formulae and most wrote this down using \(\sin(A-B)\) version. Most common error involved substitution of \(-\frac{\pi}{2}\) instead of \(\frac{\pi}{2}\).

**Question 7**
This question required working out of some description, whether it be diagram annotations or calculations. Full marks not given to just the correct solution stated. Also asked to give equation answer, and unfortunately many students stopped after identifying values for \(a\), \(b\) and \(c\). Some gave equations with horizontal translations which did not fit with \(y = a \sin(bx) + c\). However, the translations given were mostly incorrect.

**Question 8**
Credit given for students stating the basic angle as \(\frac{\pi}{4}\). Students who attempted to use general formulae or again the multiple angle formulae, often gave confused and incorrect logic throughout working out. The most common error involved the exclusion of the first or last of the three solutions.

**Section C**

**Question 9**
Quite universally well done. Part mark deduction for struggling with \(\sqrt{x}\) the most common error.

**Question 10**
Again generally well done, though a number of students only answered the first half of the question and didn’t state the nature of the roots.

**Question 11**
Probably the question candidates found the most confusing. While a significant number answered it completely, the majority were thrown by the \(\sin^2(x)\), a typical answer being 60\(\cos(5x)\). This gained them part marks.

**Question 12**
This was the other least well-answered question. Many candidates gained \(\frac{1}{2}\) mark for correctly substituting into the change of base rule and couldn’t go further. Many others went for the quotient rule and didn’t resolve it properly. Only a minority recognised that \(\ln(2)\) was a constant and proceeded from there.

**Question 13**
Many of the candidates worked their way successfully through this, although a significant number failed to deal with the common factor of \(e^x\) correctly. This, typically, attracted a penalty; however, having a more complicated substitution to find the gradient and a more complicated gradient to deal with in the equation to the line often resulted in further loss of marks.
Section D

Question 14
Generally done well. Students who used the chain rule rather than expanding were more successful with this problem. The most common errors were not making the answer negative and multiplying by 2 rather than the required dividing by 3 in the integration.

Question 15
Generally done poorly. Most students wrote the initial integration $f(x) - g(x)$ correctly but only students who simplified the expression before integrating and substituting were successful in coming to the correct answer. Otherwise, the question tended to blow out to a large numeracy problem. Part marks were awarded to students who simplified the expression prior to integrating but failed to see the double negative and therefore arrived at the incorrect answer.

Question 16
Done reasonably well generally. When integrating students tended to multiply by 3 rather than divide by 3; however, a lot of the time this is due to the fact that they don’t understand what to do when dividing a fraction by a whole number – they tend to place the 3 under the two and treat the next algebraic step as a division by a fraction. When substituting a lot of students did not recognise $\cos\left(\frac{3\pi}{6}\right) = \cos\left(\frac{\pi}{2}\right) = 0$ and again many made mistakes with the fractions on fractions.

Question 17
Mostly done well. Most students were able to use the product rule to find the derivative of $x\ln(x)$ – common error being students saying that $x$ multiplied by $1/x$ is equal to 0 rather than 1. Most common error in question is when students subtract 1 from both sides instead of the integral of $1/x$. Half mark taken off for no $+c$.

Question 18
This question was done very poorly. There were two common mistakes that occurred, the first mistake made by students was generally not dividing by 2 when integrating – this made the algebra much harder to deal with further into the solution. The second mistake was when students ignored the significance of the absolute value sign in the logarithm. Half a mark was taken off for not recognising the existence of a second solution if the $k$ was not bounded.

Section E

Question 19
A number of students just gave answers without any explanation. It was expected that some explanation or appropriately labelled diagram illustrate the stated probability. Where no explanation was given students were awarded a maximum of 2 marks. The common error in part c was to give 0.4, which is the $\Pr(13 < X < 17)$

Question 20
a) – c) Generally well done. Most students had little difficulty completing the table of outcomes and the table of probabilities. Most students knew how to calculate $E(X)$, but arithmetic errors were common in evaluating it.

b) A significant group handled this question well, but it caused some problems for a number of students who were unsure of how to interpret the information given in terms of a binomial distribution. Some students did not link this question to the remainder of Question 20. However, a number of student were partially correct and scored part marks.
Question 21
Again, a significant group handled this question well. For the remainder, common errors include not recognising this as a Hypergeometric question; and not realising that:
\[
Pr(X \geq 1) = 1 - Pr(X = 0) \quad \text{or} \quad Pr(X \geq 1) = Pr(X = 1) + Pr(X = 2) + Pr(X = 3) + Pr(X = 4) + Pr(X = 5).
\]

PART 2 – Calculators Allowed
Candidates are again reminded that calculator instructions without supporting algebra or diagrams are not working. Markers are not expected to know the different configurations for all calculators and may consider an instruction as insufficient if unfamiliar with the specific calculator or command. This was a particular issue in the probability section.

Section A

Question 22
a) Candidates generally knew the answers but were not able to express the domain and range correctly. Often large values were put before smaller ones and the wrong brackets were used. Combinations of conventions were used.

b) Most candidates could correctly find the equation for the truncus but did not justify their answers for \( h \) and \( k \) in particular. This is a 3 mark question and so working/justification was expected.

Question 23
a) Most candidates were able to plot a graph of \( f(x) = 3e^{2x} - 1 \) but were unable to solve for the exact values of intercepts. Often when exact values had been found, the approximate value for the \( x \)-intercept was labelled on the diagram. Very few related the horizontal asymptote to the vertical translation. The equation to the asymptote was sometimes given as \( x = -1 \). Some included a vertical asymptote of \( x = 1 \). The poor quality of the sketched curve was of concern.

b) Very few candidates related their answer to the sketch they produced in (a) in order to score full marks. Part marks were given for correct and logical explanations without reference to the graph in a).

Question 24
Most candidates identified \( k = 3 \) and were able to set up simultaneous equations based on the two points. However, in some cases the co-ordinates were incorrectly substituted.

Very few students were able to solve the simultaneous equations to find the values of \( a \) and \( h \). The best way to do this was with the calculator. To gain full marks, candidates needed to solve the quadratic equation to find two possible pairs of exact solutions and identify the one that fitted in the given domain, \((0,20)\). Part marks were given for the correct approximate answers.

Section B

Question 25
Most students made a start to this question and successfully identified the vertical/horizontal translations. Calculating the period was often wrong because of fraction calculation errors. The most common mistake was to substitute a value of the inflexion point, rather than the given point at \( \frac{\pi}{6} \) to find the value of \( a \).
Question 26

a) Approximately half of the marked papers had this wrong because of basic manipulation errors involving equating the tan value to 0 rather than 1.

b) Many students solved this completely on the calculator. If it was answered with detailed working it was common to see errors with fraction calculations for both the addition/subtraction and division components. The restricted domain was sometimes ignored. It was surprising to use the use of decimal values in preference to exact values.

Question 27

a) Generally very well done

b) Marks were lost frequently in this question because of not reading the wording carefully. It asked for clearly labelled maximum and minimum points which were not provided fully. The most common error was stating the maximum point as 120m, ignoring the fact that the lowest point was 1m above the ground.

c) Generally well done if part b) was well done. There were many alternative possible answers for this question. The question specifically asked for a relationship between H and T, a large number of students ignored this and wrote the equation as between y and x.

d) Most students recognised the need to equate their function to 85. Interpretation of the question was a problem as it clearly asked for the times they were able to see the park not just intersection points. Rounding of decimals was not always well done. If a time is asked for the answer should reflect actual time not just minutes elapsed. This could be improved by reading the questions more carefully.

Section C

In general, students who showed strong CAS skills tended to score highly in this section. Many students lost marks for not including a justification of the nature of a stationary point – this was expensive for some students, since every question involved a stationary point. Students who used the second derivative rather than the “sign of the derivative” test seemed to do better in this section.

Question 28

Generally well done, however mistakes in simplifying x/x to 0 after correctly applying the product rule were common. Many students lost marks for not including a y-coordinate for the stationary point or not indicating the source of their y-coordinate. Many students did not include a justification of the nature of the minimum stationary point and some lost marks for including x values that made derivative undefined or were on the same side of the stationary point. Students who included nothing other than a numerical solution for the minimum value (0.368, -0.368) were awarded 1 mark. Some students made a minor error in their differentiation, yielding a derivative that produced no stationary points. Some students stopped working at that point. Students should be encouraged to check the shape of the graph on the calculator to make sure that their answer makes sense.

Question 29

a) This question caused difficulty for a large number of students. A common mistake was the assumption that h and w were equal. This assumption prevented students from being able to derive the provided formula. Many students simply rewrote the provided formula at the end of an incomplete list of working.

b) Generally well done. The majority of students found the derivative of SA correctly and solved correctly for w. Some students stopped there. Others continued to find the minimum surface area, but lost part marks for either not including units, or not including a justification of the minimum, or both. Students were not penalised for leaving their answer in surd form.
Question 30
This question was done very well by students who were confident and capable with their CAS technology. There was some confusion as to whether the \(-(t-10)^2/100\) term was an index or a multiplier, which affected all three parts of the question. No penalty was given to students who left their answers in exact values.

a) Generally well done. Students were awarded part marks for stating that the initial value of the stock was \(P(0)\).

b) A large number of students lost marks due to errors made when transcribing the derivative to or from the calculator. The rate unit ($/week) was missed by a majority of students. Some students found the average rate of change over the first five weeks. A large number of students failed to complete the question – they found the derivative but did not find its value at \(t=5\).

c) Students who interpreted the \(-(t-10)^2/100\) term as a multiplier were still able to find a stationary point at \(t=10\), but through a much easier path than those who used the proper formula and were penalised for this. Many students failed to justify the maximum nature of the stationary point. A significant number of students substituted 10 for \(t\) in the derivative, rather than solving \(P'(t)=0\) for \(t\). This was considered to be a valid alternative. Marks were lost if insufficient working was shown before justifying the maximum. A large number of students avoided using calculus and instead used a value table for \(P(t)\) which indicated a possible peak value at \(t=10\). This incorrect approach resulted in a loss of marks.

Section D

Question 31
a) This question was done well by most students. Almost all students were able to sketch the graphs, although some didn’t adjust the view window on the calculator to show all 3 intersection points. Most students were able to label the intersection points although many also labelled axis intercepts which was not required. A small number of students sketched a parabola which was likely the results of not adjusting the view window on the calculator correctly.

b) This question was done well by most students. Some students stated the correct answer but didn’t give supporting working out in order to get the 3rd mark. A number of students also forgot to include units. Other difficulties included confusing the \(x\) and \(y\) coordinates of the intersection points when setting the upper and lower bounds and confusion when using absolute values or a negative sign in order to produce a positive resulting area. A final concern was the high number of students who attempted to work this question algebraically which was not required and put them at a significant time disadvantage.

Question 32
This question was done well by most students. The most common error was omitting the absolute value signs or negative sign required to ensure that the integral for the area below the \(x\) axis returned positive. Some students forgot to include units. Others split the graph into 5 regions which could produce a correct result but created work putting the students at a time disadvantage. A small number of students found the intersection points between graphs and used the result to calculate the area between curves, which was incorrect.

Question 33
a) This question was very well done by most students.

b) Most students could partially complete this question although many weren’t able to state a correct final answer. Most students identified that they had to integrate the velocity function. However, a very common error was interpreting “Note that \(v = \frac{ds}{dt}\) as an instruction to take the derivative. Many students didn’t include a constant when integrating and hence lost the marks assigned for finding the constant and stating the final answer. Those who included the constant were generally able to find it, although a number of students assumed that the initial displacement of 2 metres meant that the constant was 2. A handful of students who
used the calculator to integrate had their calculator in degree mode, which was not incorrect but produced a less familiar result.

c) Most students had significant difficulty with this question. The most common answer was simply substituting \( t=1 \) into the displacement expression from part b which was incorrect and could not reasonably be considered to account for 3 marks. Many students realised this and calculated the change in displacement between \( t=0 \) and \( t=1 \), however, this was not correct as the initial displacement was not the maximum positive displacement. A few students used the expression from part b to correctly determine the total change in distance, by finding the change in displacement from \( t=0 \) to the maximum displacement and then adding the change in displacement from the maximum displacement to \( t=1 \). Quite a number of students correctly identified that they could find the distance by integrating the velocity function given in the question, but most failed to account for the fact that the graph had positive and negative regions and so required a split integral. Some students tried integrating the displacement function, indicating a lack of understanding about the relationship between displacement, velocity and distance.

Section E

Question 34

This question was generally well done. Students using the Casio calculator could get both values in the one operation, however some (particularly those using TI) had to complete an additional calculation to find values using the left tail operation. Too many students used calculator notation as part of their written solution, but if a diagram was included which showed understanding, no penalty was given. Some students found the \( Z \) values as a step to calculating the \( x \) values, which was not incorrect, but used up valuable time. Some students tried to use the approximate intervals to calculate the \( x \) values and in doing so confused 90% with the 95% interval. Many students failed to add units to their final answers.

Question 35

This question was generally well done. Most students answered correctly using a hypergeometric model, however some miscalculated the total population as 90 (ie: 45 + 10 + 15 + 20) or 80 (ie: 45 + 15 + 20) or found the probability of exactly 2 blue or green marbles. Some used a binomial distribution to calculate the probability.

Question 36

a) Less than half of the students correctly identified the need for the probability of a red light to remain constant and the need for the lights to be independent in operation. Most correctly identified only one of the required factors. Many were worried by orange lights, light maintenance, car speeds, accidents etc. these could have been covered in a less specific statements. Some answers only mentioned independent and consistent probability with no relation to the situation.

b) Generally answered well with students effectively using their calculator to sum the required probabilities (binomial model), however, many read the problem as exactly 6.

c) Few students correctly used the probability calculated in (b) and correctly redefined the problem in terms of the number of days late. Some students read the problem as at least one day or exactly one day. Many simply multiplied their answer from (b) by 5.

Question 37

Many students answered this question well and were able to find 2 scores and use the equations to find and state the mean and standard deviation. Diagrams were well used by most students as an aid to calculating \( Z \) scores. However, some students tried to use the approximate intervals and used novel methods to complete the calculation. Calculator notation was present again in solutions and the final result was often missing units. A number of students did not attempt this last question on the paper.
SECTION A

Answer ALL questions in this section.

This section assesses Criteria 4.

Question 1
State a possible graph for the following function (in factorised form): (2 marks)

\[ y = a(x + 2)(x - 3)^{\frac{1}{2}} \]

\[ a \leq 0 \]

Question 2
State the domain and range of the circle with equation given by: (2 marks)

\[ (x - 5)^2 + (y + 2)^2 = 16 \]

Centre: (5, -2), Radius: 4

Domain: \[ x \in [1, 9] \]

Range: \[ y \in [-6, 2] \]

\[ -\frac{1}{2} \text{ if round brackets used} \]
Section A (continued)

Question 3

The square root graph on the left is transformed into the square root graph on the right using two reflections followed by two translations.

(a) State the two reflections required. (1 mark)

- Reflect in x-axis
- Reflect in y-axis

(b) State the two translations required. (1 mark)

- Translate 1 left
- Translate 1 down

(c) The square root graph on the left is given by \( f(x) = 2\sqrt{x} + 4 + 1 \). Find the equation of the square root graph on the right. (2 marks)

\[ \sqrt{f(x)} = -2\sqrt{3}x^2 - \frac{2}{2} \]

Section A continues.
Section A (continued)

Question 4

Two functions $f$ and $g$ are given by $f(x) = \log_3 \left( \frac{x^3}{3} \right)$ and $g(x) = \sqrt[3]{3^{x-1}}$.

(a) Show that the composite function, $f$ of $g$, is equal to $x$.
   i.e. $(f \circ g)(x) = x$.

\[
(f \circ g)(x) = \log_3 \left( \frac{\left( \frac{x^3}{3} \right)^3}{3} \right)
\]
\[= \log_3 \frac{\frac{x^9}{27}}{3}
\]
\[= \log_3 \frac{x^9}{81}
\]
\[= \log_3 \frac{x^9}{3^4}
\]
\[= \log_3 3^x
\]
\[= x
\]

(3 marks)

(b) Given that $g(f(x)) = x$ and your result in part (a), what does this say about the functions $f$ and $g$?

They are inverse functions.

(1 mark)
Answer ALL questions in this section.

This section assesses Criteria 5.

Question 5
(a) Evaluate \( \sin \left( \frac{3\pi}{2} \right) \times \cos(0) - \tan(2\pi) \times \sin \left( \frac{\pi}{2} \right) \). (2 marks)

\[
\begin{align*}
\sin \left( \frac{3\pi}{2} \right) & = -1 \\
\cos(0) & = 1 \\
\tan(2\pi) & = 0 \\
\sin \left( \frac{\pi}{2} \right) & = 1
\end{align*}
\]

\[
-1 \times 1 - 0 \times 1 = -1
\]

(b) Evaluate \( \cos(135^\circ) \times \sin(330^\circ) \). (2 marks)

\[
\begin{align*}
\cos(135^\circ) & = -\frac{\sqrt{2}}{2} \\
\sin(330^\circ) & = -\frac{1}{2}
\end{align*}
\]

\[
\left( -\frac{\sqrt{2}}{2} \right) \times \left( -\frac{1}{2} \right) = \frac{\sqrt{2}}{4}
\]

Question 6
Use the Multiple Angle Formulae to simplify \( \sin \left( x - \frac{\pi}{2} \right) \) as far as possible. (2 marks)

\[
\sin \left( x - \frac{\pi}{2} \right) = \sin x \cos \frac{\pi}{2} - \cos x \sin \frac{\pi}{2}
\]

\[
= 0 - \cos x \times 1
\]

\[
= -\cos x
\]

Section B continues.
Section B (continued)

Question 7

Find an equation that represents the graph below. Express your answer in the form
\[ y = a \sin (b x) + c \] where \( a \), \( b \) and \( c \) are real numbers. (3 marks)

\[ y = 2 \sin 3x + 3 \]
Period = \( \frac{2\pi}{3} = \frac{2\pi}{5} \)
\[ a = 2, \quad b = 3 \] (1)

\[ c = 3 \] (1)

Question 8

Solve the equation \( \cos \left( x + \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \) for \( x \in [0, 2\pi] \). (3 marks)

\[ x + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \]
\[ x = 0, \frac{3\pi}{4}, 2\pi \] (1)
Answer ALL questions in this section.

This section assesses Criteria 6.

Question 9

Find the derivative of \( f(x) = 6x^2 + 5x + 3\sqrt{x} \). 

\[
f'(x) = 12x + 5 + \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2\sqrt{x}}.
\]

(2 marks)

Question 10

A student prepares the following table for a 'change of sign of derivative test':

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>f'(x)</td>
<td>-128</td>
<td>0</td>
<td>24</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>-200</td>
</tr>
</tbody>
</table>

Give the \( x \) coordinate of the stationary points of \( f(x) \) and state their nature.  

local min @ \( x = -1 \) 

horizontal point of inflection @ \( x = 2 \) 

local max @ \( x = 6 \)  

(3 marks)
Section C (continued)

Question 11

Find the derivative of \( y = 6 \sin^2(5x) \). (2 marks)

\[
\frac{dy}{dx} = 12 \sin(5x) \cdot 5 \cos(5x) = 60 \sin(5x) \cos(5x) = 30 \sin(10x)
\]

Question 12

Using the change of base rule for logarithms, \( \log_a B = \frac{\log_c B}{\log_c a} \).

Find the derivative of \( y = \log_2(3x) \). (2 marks)

\[
\frac{dy}{dx} = \frac{\ln 3}{\ln 2} \cdot \frac{3}{x} = \frac{3}{x \ln 2}
\]

Question 13

Find the tangent to the equation \( f(x) = \frac{x^2}{e^x} \) at the point \( \left( 1, \frac{1}{e} \right) \). (3 marks)

\[
f'(x) = \frac{2x e^x - e^x x^2}{(e^x)^2} = \frac{2x - x^2}{e^x}
\]

\[
f'(1) = \frac{1}{e} - \frac{1}{e} = \frac{1}{e}
\]

\[
f'(1) = \frac{\ln(2 - x)}{(e^x)^2} = \frac{e^x}{x(2 - x)}
\]

\[
\frac{dy}{dx} = \frac{\ln(2 - x)}{(e^x)^2} = \frac{e^x}{x(2 - x)}
\]

\[
f'(1) = \frac{1}{e} - \frac{1}{e} = \frac{1}{e}
\]

\[
y = \frac{\ln(2 - x)}{e} + C
\]

\[
\frac{e}{e} = \frac{1}{e} + C
\]

\[
\therefore y = \frac{x}{e}
\]
Answer ALL questions in this section.

This section assesses Criteria 7.

**Question 14**

Find \( \int \frac{(3-x)^2}{5} \, dx \) \hspace{1cm} (2 marks)

\[ -\frac{(3-x)^3}{15} + C \]

\[ -\frac{1}{2}, \text{ no units} \]

**Question 15**

The two functions \( f(x) = x^2 + 3x + 4 \) and \( g(x) = x^2 + 3x - 2 \) are drawn below for \( x \in [-1, 3] \).

Find the area between the two curves between \( x = -1 \) and \( x = 3 \). \hspace{1cm} (2 marks)

\[ \text{Area} = \int_{-1}^{3} (x^2 + 3x + 4) - (x^2 + 3x - 2) \, dx \]

\[ = \int_{-1}^{3} (6x + 6) \, dx \]

\[ = [6x^2]_{-1}^{3} \]

\[ = 18 - (-6) \]

\[ = 24 \, \text{units}^2 \]

\[ -\frac{1}{2}, \text{ no units} \]
Section D (continued)

Question 16
Determine the value of \( \int_{\pi/6}^{\pi/4} \frac{\sin(3x)}{2} \, dx \). (2 marks)

\[ \frac{1}{2} \int_{\pi/6}^{\pi/4} -\frac{1}{3} \cos 3x \, dx = -\frac{1}{6} \left( \cos \frac{3\pi}{4} - \cos \frac{3\pi}{6} \right) \]

\( = -\frac{1}{6} \left( -\frac{\sqrt{2}}{2} - 0 \right) \)

\[ = \frac{\sqrt{2}}{12} \] (1)

Question 17
Differentiate \( y = x \ln x \) and use this result to find \( \int \ln x \, dx \). (3 marks)

\[ \frac{dy}{dx} = 1 \ln x + \frac{1}{x} \cdot x \]

\[ = \ln x + 1 \]

\[ \int \ln x \, dx = \int \ln x + 1 \, dx - \int 1 \, dx \]

\[ = x \ln x - x + C \]

\[ -\frac{1}{2} \text{ for } C \] (1)

Question 18
Find \( k \) such that \( \int_{-2}^{k} \frac{3}{2x-1} \, dx = \frac{3 \ln 5}{2} \), for \( k > 1 \). (3 marks)

\[ \int_{-2}^{k} \frac{3}{2x-1} \, dx = \frac{3}{2} \ln |2x-1| \]

\[ \frac{3}{2} \ln 5 = \frac{3}{2} \ln (2k-1) \]

\[ 2k-1 = 5 \]

\[ k = 3 \] or \( 2 \)

but \( k > 1 \) : \( k = 3 \) (1)

\[ -\frac{1}{2} \text{ if } -2 \text{ were eliminated of } \]

11 not explained
SECTION E

Answer **ALL** questions in this section.

This section assesses **Criteria 8**.

**Question 19**

The length of leaves on a tree is found to fit a normal distribution $X$ with mean 15cm.

Given that the probability of a leaf being 17cm or longer is 0.3 (i.e. $\Pr(X \geq 17) = 0.3$)

(3 marks)

[Diagram showing normal distribution with mean 15 and some probabilities]  

Calculate:

(a) $\Pr(X \leq 17)$  

$\phantom{0.7}$

(b) $\Pr(X \leq 13)$  

$\phantom{0.3}$

(c) $\Pr(13 \leq X \leq 15)$  

$\phantom{0.2}$

Answers only = 2 marks

Section E continues.
Section E (continued)

Question 20

A spinner with sectors marked 1 to 4 is spun twice and the greater of the two results recorded.

For example if 2 and 3 are spun, 3 is recorded, if 4 and 4 are spun, 4 is recorded.

(a) Complete the following table of possible outcomes. (1 mark)

<table>
<thead>
<tr>
<th>First Spin</th>
<th>Second Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1  2  3  4</td>
</tr>
<tr>
<td>1</td>
<td>1  2  3  4</td>
</tr>
<tr>
<td>2</td>
<td>2  2  3  4</td>
</tr>
<tr>
<td>3</td>
<td>3  3  3  4</td>
</tr>
<tr>
<td>4</td>
<td>4  4  4  4</td>
</tr>
</tbody>
</table>

(b) Complete the probability distribution table for the distribution X. Leave your answers as fractions. (1 mark)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob</td>
<td>1/16</td>
<td>3/16</td>
<td>5/16</td>
<td>7/16</td>
</tr>
</tbody>
</table>

(c) Calculate the Expected Value, E(X), of the distribution in part (b). (2 marks)

\[ E(X) = \frac{1}{16} + \frac{6}{16} + \frac{15}{16} + \frac{28}{16} \]

\[ = \frac{50}{16} = \frac{25}{8} \approx 3.125 \]

Question 20 continues.
Question 20 (continued)
(d) A person uses the spinner to obtain 10 results. What is an expression for the probability that 4 of the results are 3?

Leave your answer in the form \[ C_n(p)^r(1-p)^{n-r}. \]  (2 marks)

\[
P(X = 4) = \binom{10}{4} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^6
\]

Question 21

The Senate of a country consists of 45 men and 65 women.

5 Senators are to be selected at random to form a committee.

If \( X \) is the random variable for the number of men in the committee, what is \( \Pr(X \geq 1) \), i.e. the probability that the committee includes at least one man?

Note that \( C_n \) terms can be left not simplified in your answer.

\[
\Pr(X \geq 1) = 1 - \Pr(X = 0) = \binom{110}{0} \left(\frac{45}{110}\right)^0 \left(\frac{65}{110}\right)^{110}
\]

\[
= 1 - \frac{\binom{45}{5} \binom{65}{5}}{\binom{110}{5}}
\]

Defining \( N \) as in worth 1 mark.
Answer ALL questions in this section.

This section assesses Criteria 4.

**Question 22**

(a) State the domain and range of the truncus pictured above. (1 mark)

Domain: $x \neq 2$  

Range: $y < 5$

(b) Find an equation that represents the truncus pictured above. (3 marks)

$y = \frac{(x-2)^2}{(1-2)^2} + 5$ because asymptote at $x=2$ and $y=5$

$2 = \frac{a}{(1-2)^2} + 5$

$\therefore a = -3$  

$y = \frac{-3}{(x-2)^2} + 5$

Section A continues.
Consider the function \( f(x) = 3e^{2x} - 1 \).

(a) Sketch the graph of the function on the axes below. Include the equation for the asymptote as well as exact values for the \( x \) and \( y \) intercepts in your sketch.

\[
\begin{align*}
f(0) &= 3e^0 - 1 \\
&= 2 \\
\frac{1}{3} &= e^{2x} \\
\frac{1}{2} &= x = -\ln 3 \\
&= -1 \frac{1}{2}
\end{align*}
\]

Translated 1 down: \( y = -1 \), asympt \( \frac{1}{2} \).

(b) Using your sketch above, determine the number of solutions of the equation

\[3e^{2x} + x^2 = 0.\]

\[3e^{2x} - 1 = -x^2 - 1.\]

\[\therefore \text{ there are no sol's (see graph).}\]
A teenager is standing with a toy that fires foam bullets at one end of a long hallway and hits the target at the other end.

He is firing from a height of 1m and the target is at a height of 2m. The ceiling in the hall is at a height of 3m and so the maximum height of the bullet will be 3m.

Find an equation for the path of the foam bullet in the form \( y = a(x-h)^2 + k \).

Give exact values for \( a \), \( h \), and \( k \) in your answer. (6 marks)

\[
\begin{align*}
\text{max height } 3m & \Rightarrow k = 3 \quad \therefore y = a(x-h)^2 + 3 \\
(0,1) & \Rightarrow 1 = a(0-h)^2 + 3 \\
& \quad \therefore 1 = ah^2 + 3 \quad \therefore ah^2 = -2 \quad \textbf{(1)} \\
(20,2) & \Rightarrow 2 = a(20-h)^2 + 3 \\
& \quad = a(400 - 40h + h^2) + 3 \\
& \quad = a(400 - 40h + ah^2) + 3 \quad \textbf{(2)} \\
\text{from calc} \quad & a = -\frac{1600}{52} + 2400 \quad \therefore h = -20.5 + 40 \\
& a = 1600 \frac{52}{2} + 2400 \quad h = 20.5 + 40 \\
\text{from graph} \quad & a < 0 \quad \& \quad 0 \leq h < 20 \\
& a = -\frac{1600}{52} + 2400 \quad h = 40 - 20.5 \\
\end{align*}
\]
Answer ALL questions in this section.

This section assesses Criteria 5.

Question 25

Give an equation for $y$ in terms of $x$ in the form $y = a \tan(n(x + b)) + c$, for $a$, $b$, $c$, $n$ real numbers.

Period = $\frac{\pi}{2} = \frac{\pi}{n}$ $\therefore n = 2$

no translations left/right $\therefore b = 0$

trans up $\therefore c = 1$

$y = a \tan 2x + 1$

$2\sqrt{3} + 1 = a \tan \frac{2\pi}{6} + 1$

$2\sqrt{3} + 1 = a \sqrt{3} + 1$

$\therefore a = 2$ and $y = 2 \tan 2x + 1$

(1)

Section B continues.
Section B (continued)

Question 26

Given the equation $\sqrt{3} \sin\left(2x + \frac{\pi}{4}\right) = \cos\left(2x + \frac{\pi}{4}\right)$.

(a) Write this equation in the form $\tan\left(nx + b\right) = c$. (1 mark)

\[ \tan\left(2x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \]

(b) Find all solutions of this equation for $x \in [\pi, 3\pi]$. (2 marks)

\[
2x + \frac{\pi}{4} = \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{19\pi}{12}, \frac{25\pi}{12}, \frac{31\pi}{12}, \frac{34\pi}{12}, \frac{37\pi}{12}, \frac{41\pi}{12}, \frac{47\pi}{12}, \frac{51\pi}{12}, \frac{55\pi}{12}, \frac{61\pi}{12}, \frac{65\pi}{12}, \frac{71\pi}{12}
\]

\[ 2x = \frac{7\pi}{12} - \frac{\pi}{4}, \frac{3\pi}{4} - \frac{\pi}{4}, \frac{19\pi}{12} - \frac{\pi}{4}, \frac{25\pi}{12} - \frac{\pi}{4}, \frac{31\pi}{12} - \frac{\pi}{4}, \frac{34\pi}{12} - \frac{\pi}{4}, \frac{37\pi}{12} - \frac{\pi}{4}, \frac{41\pi}{12} - \frac{\pi}{4}, \frac{47\pi}{12} - \frac{\pi}{4}, \frac{51\pi}{12} - \frac{\pi}{4}, \frac{55\pi}{12} - \frac{\pi}{4}, \frac{61\pi}{12} - \frac{\pi}{4}, \frac{65\pi}{12} - \frac{\pi}{4}, \frac{71\pi}{12} - \frac{\pi}{4} \]

\[ x = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{19\pi}{24}, \frac{25\pi}{24}, \frac{31\pi}{24}, \frac{34\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}, \frac{47\pi}{24}, \frac{51\pi}{24}, \frac{55\pi}{24}, \frac{61\pi}{24}, \frac{65\pi}{24}, \frac{71\pi}{24} \]
Section B (continued)

Question 27

The Melbourne Star is a giant ferris wheel that has a diameter of 120m, and at its lowest point is 1m above the ground.

The ferris wheel takes 30 minutes to make one revolution without stopping.

(a) A tourist enters a carriage at the bottom of the wheel at 12pm and stays on the wheel for two revolutions. At what times will they be at the top of the wheel? (1 mark)

12:15pm, 12:45pm

(b) Using the grid below, sketch a graph of height of the carriage above the ground, H, against time, t, for the 60 minutes from 12pm. Clearly label the maximum and minimum points on your graph. (2 marks)

Question 27 continues.
Question 27 (continued)

(c) Write a suitable equation that models the relationship between \( H \) and \( t \) for two revolutions.

\[
\text{middle: } \frac{121+1}{2} : y = 61 \quad \text{amp: } 60 \quad \frac{1}{2} \\
\text{start: } \min \Rightarrow y = -60 \cos 15x + 61 \quad \frac{1}{2} \\
\text{Period: } 30 = \frac{\pi}{b} \quad \frac{1}{2} \\
\therefore b = \frac{b}{15} \quad 0 = -60 \cos \frac{\pi x}{15} + 61 \quad \frac{1}{2} \\
\frac{1}{2} \quad y, x
\]

(d) When the carriage is 85m or more above the ground they are able to see Luna Park in St Kilda. Calculate, to the nearest minute, the times they are able to see Luna Park.

(3 marks)

\[
\text{see graph for points of intersection} \quad \frac{2}{2} \\
\text{between 12:09 and 12:11pm} \quad \frac{1}{2} \\
\text{and between 12:39 pm to 12:41pm} \quad \frac{1}{2}
\]
Answer ALL questions in this section.

This section assesses Criteria 6.

**Question 28**

Find and classify any stationary points for \( f(x) = x \ln(x) \), where \( x > 0 \).  

\[
\frac{d}{dx}f(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} \\
0 = \ln(x) + 1 \\
-1 = \ln(x) \\
\therefore x = e^{-1} \\
f(e^{-1}) = \frac{1}{e} \cdot \ln(e^{-1}) = -\frac{1}{e} \\
\begin{array}{c|c|c}
 x & f(x) & f'(x) \\
-\frac{1}{e} & 0 & 1 \\
\end{array} \\
\text{local min} @ (e^{-1}, -\frac{1}{e})
Section C (continued)

Question 29

A chocolate manufacturer wishes to produce a chocolate bar called the 'Chocolate Stick'. The bar is to contain a volume of 100 cm³ of chocolate.

Its length is to be \( l = 20 \text{ cm} \).

The amount of wrapping to be used needs to be as little as possible so the surface area of the bar must be minimised.

(a) Show that the surface area is given by \( S = 40w + 10 + \frac{200}{w} \), where \( w \) is the width of the bar. (2 marks)

\[
S = 2hw + 2 \times 20w + 2 \times 20h = \frac{1}{2} w^2 + 10w + \frac{200}{w} \quad \text{since} \quad V = 20h \times w
\]

\[
= 10 + 40w + \frac{200}{w} \quad \text{so} \quad 100 = 20h \times w \Rightarrow h = \frac{w}{20}
\]

(b) Hence find the minimum surface area of the chocolate bar. (4 marks)

\[
\frac{dS}{dw} = 40 - \frac{200}{w^2}
\]

\[
0 = 40 - \frac{200}{w^2}
\]

\[
\frac{200}{w^2} = 40 \Rightarrow w^2 = 5
\]

\[
w = \sqrt{5} \quad \text{(since} \quad w > 0 \text{)}
\]

\[
\text{Area} = 10 + 40\sqrt{5} + \frac{200}{\sqrt{5}}
\]

\[
= 188.89 \text{ cm}^2
\]

\[
\frac{dS}{dw} = -\frac{200}{w^3} \quad \text{so} \quad \text{minimum at} \quad w = \sqrt{5}
\]

Section C continues.
Section C (continued)

Question 30

A new stock is to be listed for purchase and trade on the stock exchange. An economist predicts that the share price will follow the function \( P(t) = 3e^{-\frac{(t-10)^2}{100}} \), where \( P \) is the price in dollars and \( t \) is the time in weeks.

(a) Find the initial value of the stock. (1 mark)

\[
P(0) = 3e^{-\frac{(0-10)^2}{100}}
\]

\[
= 3e^{-\frac{100}{100}} \\
= 3e^{-1} \\
= \frac{3}{e} \quad \text{(accepted)}
\]

(b) Find the rate of change of the stock after 5 weeks.

You may use your calculator to assist in finding the derivative here. (2 marks)

\[
P'(t) = -6t e^{-\frac{(t-10)^2}{100}}
\]

\[
P'(5) = -6(5) e^{-\frac{(5-10)^2}{100}}
\]

\[
= -30 e^{-\frac{25}{100}}
\]

\[
\approx 0.234 \quad \text{or} \quad 0.234 \text{ per week (exact accepted)}
\]

(c) Show using calculus that the stock will reach a maximum price after 10 weeks. (3 marks)

\[
0 = -3(x-10)e^{-\frac{(x-10)^2}{100}}
\]

\[
\text{since } e^x > 0 \quad \text{need } x-10 = 0 \quad \Rightarrow x = 10
\]

\[
\frac{dP}{dt} |_{x=10} = 0 \quad \text{local max @ } x = 10
\]

\[
\text{or after 10 weeks}
\]
Answer ALL questions in this section.

This section assesses Criteria 7.

Question 31

(a) Sketch the graph of the two equations \( f(x) = x^3 - 3x^2 - 4x + 9 \) and \( g(x) = 2x + 1 \) including the intersections points. (2 marks)

(b) Find the area enclosed between the two curves \( f(x) \) and \( g(x) \). (3 marks)

\[
\text{Area 1} = \int_{-2}^{1} (x^3 - 3x^2 - 4x + 9) - (2x + 1) \, dx = \frac{81}{4}
\]

\[
\text{Area 2} = \int_{1}^{4} (3x^2 + 6x - 8 - x^3) \, dx = \frac{81}{4}
\]

\[
\therefore \text{Total area} = \frac{81}{2} \text{ or } 40.5 \text{ sq units}
\]
Section D (continued)

Question 32

A shaded region is constructed from the three parabolas as shown below:

\[ f(x) = -\frac{1}{3}(x+3)(x-1), \]
\[ g(x) = \frac{1}{10}(x+1)(x-8), \text{ and} \]
\[ h(x) = -(x-7)(x-9). \]

Find the area enclosed between the three parabolas and the x-axis.

Note: You may use your calculator, but an expression involving the integrals concerned must be included in your answer. (4 marks)

\[
\int_{-3}^{\infty} -\frac{1}{3}(x+3)(x-1)\,dx = \frac{32}{9} 
\]

\[
\int_{-1}^{8} \frac{1}{10}(x+1)(x-8)\,dx = \frac{243}{20} \quad \text{(Area below axis)}
\]

\[
\int_{7}^{9} -(x-7)(x-9)\,dx = \frac{4}{3}
\]

\[
\text{Area} = \frac{32}{9} + \frac{243}{20} + \frac{4}{3} = 17.04 \text{ u}^2 \quad \text{(exact)}
\]

\[(2) = \text{ans only}\]

Section D continues.
Section D (continued)

Question 33

A particle has a velocity function given by \( v = 6 \cos(3t) + ke^{-2t} \), where \( k \) is a constant and \( t \) is the time in seconds.

It is known that the particle is at rest at 0 seconds.

(a) Find the value of \( k \). (1 mark)

\[
\begin{align*}
\dot{v}_0 &= 0 = 6 \cos(0) + ke^0 \\
&= 6 + k \\
\therefore k &= -6
\end{align*}
\]

(b) Find an expression for the displacement for the particle, \( s \), in metres in terms of \( t \), given that the initial displacement of the particle is 2 metres.

Note that \( v = \frac{ds}{dt} \). (3 marks)

\[
\begin{align*}
\int (6 \cos(3t) - 6e^{-2t})dt &= \frac{2}{3} \sin 3t + \frac{3}{2} e^{-2t} + C \\
2 &= 2 \sin(0) + 3e^0 + C \\
&= 3 + C \\
\therefore C &= -1
\end{align*}
\]

\[
\begin{align*}
s &= 2 \sin 3t + \frac{3}{2} e^{-2t} - 1
\end{align*}
\]

(c) Find the total distance travelled by the particle in the first second. (3 marks)

Give your answer to 3 decimal places.

\[
\begin{align*}
\text{distance} &= \int_0^{0.3506} \sqrt{v^2} \, dt + \int_{0.3506}^{1.0311} \sqrt{v^2} \, dt \\
&= 2.761 \text{ m}
\end{align*}
\]
SECTION E

Answer ALL questions in this section.

This section assesses Criteria 8.

**Question 34**

The length of nails produced by a machine are normally distributed with a mean of 100mm and a standard deviation of 1.2mm.

Determine the values $x_1$ and $x_2$, centred around the mean, such that the makers of the nails can be 90% confident that a given nail lies between these values.

\[ \Pr(x_1 \leq X \leq x_2) = 0.90 \]

(3 marks)

\[ \Pr(x \leq x_1) = 0.05 \]

\[ x_1 = 98.03 \text{mm} \]

\[ x_2 = 100 + (100 - 98.03) \]

\[ = 101.97 \text{mm} \]

**Question 35**

A bag contains 45 marbles, 10 red, 15 green and 20 blue. 6 are to be drawn out at random without replacement.

Calculate the probability of drawing exactly 2 red marbles.

(2 marks)

\[ \Pr(2 \text{ red}) = \frac{\binom{10}{2} \binom{35}{4}}{\binom{45}{6}} \]

\[ = 0.2893 \quad (\text{exact}) \]

2 marks only

Section E continues.
Section E (continued)

Question 36

A commuter has to drive through 8 sets of traffic lights on their way to work. They would like to model the number of red traffic lights they have to stop at with a binomial distribution.

(6 marks)

(a) What are two of the assumptions that need to be made before the binomial probability distribution can be used in this situation?

• Every set of traffic lights has the same probability of being red

\[ \text{Pr}(\text{being red}) \]

• The probability of each set of lights being red is independent of the others

\[ \frac{1}{2} \text{ each independent } \frac{1}{2} \text{ for situation} \]

(b) The commuter is satisfied they can use the binomial distribution. If the probability of each traffic light being red is 0.4, what is the probability of at least 6 traffic lights being red on the way to work?

\[ X = \# \text{ red lights} \quad X \sim \text{Bi}(8, 0.4) \]

\[ \text{Pr}(X \geq 6) = 0.0498 \]

(c) The commuter knows that if at least 6 traffic lights are red on a given day they will be late for work.

What is the probability they will be late for work more than once in a single 5 day working week?

\[ Y = \# \text{ days late} \quad Y \sim \text{Bi}(5, 0.0498) \]

\[ \text{Pr}(Y > 1) = \text{Pr}(Y \geq 2) = 0.0224 \]

\[ 2 \quad \frac{1}{2} \]

Section E continues.
Section E (continued)

Question 37

A camp is being held for pole vaulters in the lead up to the Olympics. The top 5%, who can jump a height of 5.80m or more, will receive automatic selection. The bottom 10%, who cannot jump above a height of 5.20m, will be sent home.

Assuming the heights which the pole vaulters can jump is normally distributed, find the mean and standard deviation of this distribution. (5 marks)

\[ Pr(X \geq 5.80) = 0.05 \quad Pr(X \leq 5.20) = 0.10 \]

\[ Pr(Z \geq z_1) = 0.05 \quad Pr(Z \leq z_2) = 0.10 \]

\[ z_1 = 1.64 = \frac{5.80 - \mu}{\sigma} \quad z_2 = -1.28 = \frac{5.20 - \mu}{\sigma} \]

\[ \text{From calc.} \quad \mu = 5.46 \text{cm, } \sigma = 0.21 \text{cm} \]